In-Medium Pion Valence Distributions in a Light-Front Model

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Abstract

Pion valence distributions in nuclear medium and vacuum are studied in a light-front constituent quark model. The in-medium input for studying the pion properties is calculated by the quark-meson coupling model. We find that the in-medium pion valence distribution, as well as the in-medium pion valence wave function, are substantially modified at normal nuclear matter density, due to the reduction in the pion decay constant.

Key words: Pion, Nuclear Medium, Light-Front, Pion Distribution Amplitude, Parton Distribution

Introduction: Light-front constituent quark model (LFCQM) has been very successful in describing the hadronic properties in vacuum, in particular, the electromagnetic form factors, electromagnetic radii and decay constants of pion and kaon [1,2,3,4,5,6]. On the other hand, recent advances in experiments make it possible to access to these hadronic properties in nuclear medium [7,8,9,10,11].

Among the all hadrons, pion is the lightest, and it is believed as a Nambu-Goldstone boson, which is realized in nature emerged by the spontaneous breaking of chiral symmetry. This Nambu-Goldstone boson, pion, plays very important and special roles in hadronic and nuclear physics [12,13,14,15,16,17,18,19,20,21,22,23].

However, because of its special properties, particularly the unusually light mass, it is not easy to describe the pion properties in medium as well as in vacuum based on naive quark models, even though such models can be successful in describing other hadrons. Recently, we studied the properties of pion in nuclear medium [10,11], namely, the electromagnetic form factor, charge radius and weak decay constant, using a light-front constituent quark model. There, the in-medium input was calculated by the quark-meson coupling (QMC) model [7,24]. We have predicted the in-medium changes of pion properties [10,11]: (i) faster falloff of the pion charge form factor as increasing the negative of the four-momentum transfer squared, (ii) increasing of the root mean-square radius as increasing nuclear density, and (iii) decreasing of the decay constant as increasing nuclear density. In this work, we extend our work for the pion in medium [10,11], and study the pion valence distribution amplitude, and parton distribution function in symmetric nuclear matter. We find a substantial modification of the pion valence wave function and distribution amplitude in nuclear medium at normal nuclear matter density.

The Model: The light-front constituent quark model (LFCQM) we use here [3,4] is quite successful in describing the properties of pion in vacuum, such as the electromagnetic form factor, charge radius and weak decay constant. The model was also extended for kaon in Ref. [5]. This fact is a prerequisite to study the in-medium changes of pion and kaon properties. In this study, we focus on the pion. For some in-medium properties of pion studied, see Ref. [10]. Note that, we simply use the terminology *medium* or *nuclear medium* hereafter, instead of explicitly specifying *symmetric nuclear matter*, otherwise stated.

In order to study the in-medium pion properties, we use the input calculated by the quark-meson coupling (QMC) model [7], the same as that already done in Ref. [10]. The QMC model was invented by Guichon [24] to describe the nuclear matter based on the quark degrees of freedom. The self-consistent exchange of the scalar-isoscalar σ and vector-isoscalar ω mean fields coupled directly to the relativistic confined quarks, are the key and novelty for the new saturation mechanism of nuclear matter. The model was extended, and has successfully been applied for various nuclear and hadronic phenomena [7]. In the following we briefly summarize the input used for the present study of the pion properties in nuclear medium.

The mass of the light quarks (q and $\bar{q}$, with $q = u, d$) in the light-front constituent quark model in vacuum [10] is, $m_q = m_{\bar{q}} = 220$ MeV. Then, all the nuclear matter saturation properties are generated by using this light quark mass value. In other words, the different values of m_q in vacuum generate the corresponding different nuclear matter properties, except for the saturation point of the symmetric nuclear matter, $\rho = \rho_0$ (normal nuclear matter density) with the empirically extracted binding energy of 15.7 MeV. This saturation point condition is generally used to constrain the models of nuclear matter.

As an example, we show in Fig. 1 negative of the binding energy, $E^{tot}/A - m_N$, for symmetric nuclear matter calculated by the QMC model with the vacuum value $m_q = m_{\bar{q}} = 220$ MeV. The corresponding incompressibility K obtained

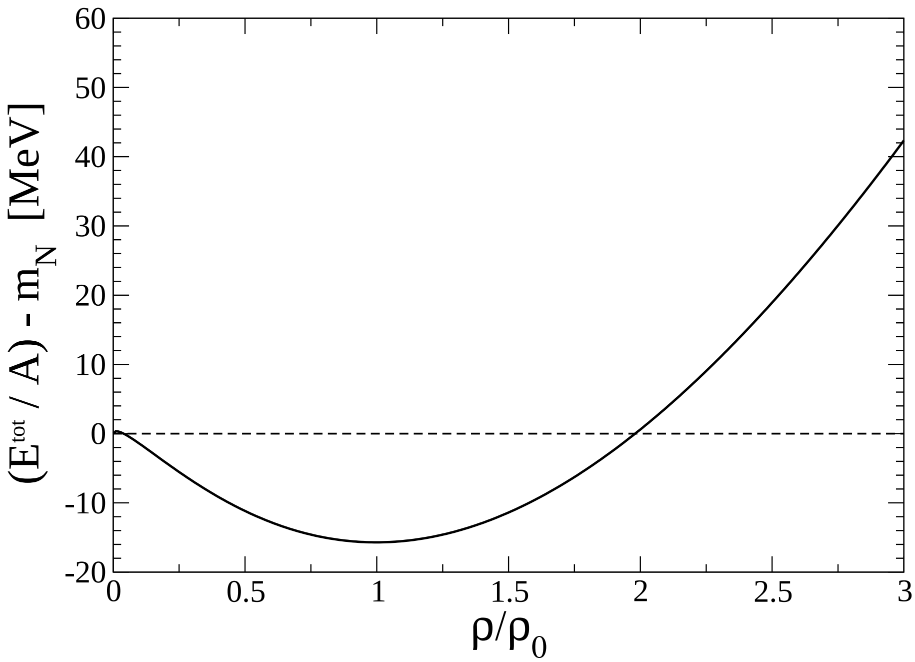

Fig. 1. Negative of the binding energy per nucleon for symmetric nuclear matter, $(E^{tot}/A) - m_N$, as a function of nuclear density ρ ($\rho_0 = 0.15$ fm^{-3}) with the quark mass value $m_q = m_{\bar{q}} = 220$ MeV ($q = u, d$), calculated by the QMC model. The corresponding incompressibility K obtained is $K = 320.9$ MeV.

is $K = 320.9$ MeV. For the other quantities calculated with $m_q = m_{\bar{q}} = 220$ MeV, see Ref. [10]. Here, we note that the pion mass up to normal nuclear matter density is expected to be modified only slightly, where the modification δm_π at nuclear density $\rho = 0.17$ fm^{-3}, averaged over the pion isospin states, is estimated as $\delta m_\pi \simeq +3$ MeV [8,25,26,27]. Therefore, we approximate the effective pion mass value in nuclear medium to be the same as in vacuum, $m_\pi^* \simeq m_\pi$, up to $\rho = \rho_0 = 0.15$ fm^{-3}, the maximum nuclear matter density treated in this study. Furthermore, since the light-front constituent quark model is rather simple, and based on a naive constituent quark picture, the model cannot discuss the chiral limit of vanishing (effective) light quark mass.

We next study the pion properties in symmetric nuclear matter with the input calculated by the QMC model. The effective interaction Lagrangian density for the quarks and pion in medium is given by,

$$\mathcal{L}_{\text{eff}} = -ig^* \left(\bar{q}\gamma^5 \vec{\tau} q \cdot \vec{\phi}\right) \Lambda^* \ , \tag{1}$$

where the coupling constant, $g^* = m_q^*/f_\pi^*$, is obtained by the "in-medium Goldberger-Treiman relation" at the quark level, with m_q^* and f_π^* being respectively the effective constituent quark mass and pion decay constant in medium, $\vec{\phi}$ the pion field [3,4,5], and Λ^* is the π-q-$\bar{q}$ vertex function in medium. Hereafter, the in-medium quantities are indicated with the asterisk, *.

Symmetric pion valence wave function: The pion valence wave function used in this study, to calculate the pion distribution amplitude (PDA) [28,29] and parton distribution function (PDF) [30,31], is symmetric under the exchange of quark and antiquark momenta. This π-q-$\bar{q}$ vertex function, Λ in vacuum, is the same as that used for studying the properties of pion [3,4,32] and kaon [5]. But for the in-medium Λ^*, the arguments of the function are replaced by those of the in-medium. The Bethe-Salpeter amplitude in medium, Ψ^*_π, with the vertex function in medium Λ^* is given by,

$$\Psi^*_\pi(k+V,P) = \frac{\not{k}+\not{V}+m^*_q}{(k+V)^2 - m^{*2}_q + i\epsilon}\gamma^5\Lambda^*(k+V,P) \times \frac{\not{k}+\not{V}-\not{P}+m^*_q}{(k+V-P)^2 - m^{*2}_q + i\epsilon}, \quad (2)$$

where $V^\mu = \delta^\mu_0 V^0$ is the vector potential felt by the light quarks in the pion immersed in medium, and can be eliminated by the variable change in the k-integration, $k^\mu + \delta^\mu_0 V^0 \to k^\mu$. Furthermore, by eliminating the instantaneous terms, namely eliminating the terms with gamma matrix γ^+ in the numerators and k^+ and $(P^+ - k^+)$ in the denominators with the light-front convention $a^\pm \equiv a^0 \pm a^3$, and integrating over the light-front energy k^-, we obtain the in-medium pion valence wave function Φ^*_π,

$$\Phi^*_\pi(k^+,\vec{k}_\perp;P^+,\vec{P}_\perp) = \frac{P^+}{m^{*2}_\pi - M^2_0}\left[\frac{N^*}{(1-x)(m^{*2}_\pi - \mathcal{M}^2(m^{*2}_q, m^2_R))} + \frac{N^*}{x(m^{*2}_\pi - \mathcal{M}^2(m^2_R, m^{*2}_q))}\right], \quad (3)$$

where, $N^* = C^*(m^*_q/f^*_\pi)(N_C)^{\frac{1}{2}}$ (N_c the number of colors), is the normalization factor [3,4,10], and $x = k^+/P^+$ with $0 \le x \le 1$; $\mathcal{M}^2(m^2_a, m^2_b) = \frac{\vec{k}^2_\perp + m^2_a}{x} + \frac{(\vec{P}-\vec{k})^2_\perp + m^2_b}{1-x} - \vec{P}^2_\perp$, and the square of the mass M^2_0, is $M^2_0 = \mathcal{M}^2(m^{*2}_q, m^{*2}_q)$. For the details of how the in-medium pion valence wave function is obtained, see Refs. [3,10]. Note that the model used in Refs. [2,33] does not have the second term in Eq. (3). This means that the pion valence wave function in Refs. [2,33] is not symmetric under the exchange of quark and antiquark momenta.

The present model with the symmetric vertex [3,4,5,32], was demonstrated successful in describing the pion properties in nuclear medium [10,11]. The pion transverse momentum probability density in medium, $P^*_\pi(k_\perp)$, in the pion rest frame $P^+ = m^*_\pi$ is calculated by,

Table 1
Properties of pion in medium, taken from Ref. [10], with $\rho_0 = 0.15$ fm^{-3}.

ρ/ρ_0	m_q^* [MeV]	f_π^* [MeV]	$< r_\pi^{*2} >^{1/2}$ [fm]	η^*
0.00	220	93.1	0.73	0.782
0.25	179.9	80.6	0.84	0.812
0.50	143.2	68.0	1.00	0.843
0.75	109.8	55.1	1.26	0.878
1.00	79.5	40.2	1.96	0.930

$$P_\pi^*(k_\perp) = \frac{1}{4\pi^3 m_\pi^*} \int_0^{2\pi} d\phi \int_0^{m_\pi^*} \frac{dk^+ M_0^{*2}}{k^+(m_\pi^* - k^+)} |\Phi_\pi^*(k^+, \vec{k}_\perp; m_\pi^*, \vec{0})|^2, \tag{4}$$

and the integration over $k_\perp$ for $P_\pi^*(k_\perp)$ leads to the in-medium probability of the valence component in the pion, η^* [3,4,10]:

$$\eta^* = \int_0^\infty dk_\perp k_\perp P_\pi^*(k_\perp). \tag{5}$$

The pion decay constant in medium in terms of the pion valence component is obtained by integrating $\Phi_\pi^*(k^+, \vec{k}_\perp; m_\pi^*, \vec{0})$ over the light-front energy k^- [3,10]:

$$f_\pi^* = \frac{m_q^*(N_c)^{1/2}}{4\pi^3} \int \frac{d^2k_\perp dk^+}{k^+(m_\pi^* - k^+)} \Phi_\pi^*(k^+, \vec{k}_\perp; m_\pi^*, \vec{0}). \tag{6}$$

Some properties of the pion in symmetric nuclear matter obtained [10], are summarized in table 1.

The results listed in table 1 are summarized as follows. As the nuclear density increases, the in-medium effective constituent quark mass, m_q^*, and the pion decay constant, f_π^*, decrease, while the square root mean charge radius, $< r_\pi^{*2} >^{1/2}$, and the probability of valence component in the pion, η^*, increase. This can be understood as follows. The reduction in mass, m_q^*, makes easier to excite the valence quark component in the pion, and resulting to increase the valence component probability η^* in the pion. Furthermore, the valence wave function spreads more in the coordinate space by the decrease of m_q^*, and reduces the absolute value of the wave function at the origin ($f_\pi^* \propto |\Phi_\pi^*(\vec{r} = \vec{0})|$ reduction [34]), namely, increases $< r_\pi^{*2} >^{1/2}$.

In-medium pion Distribution Amplitude: Pion distribution amplitude (PDA)

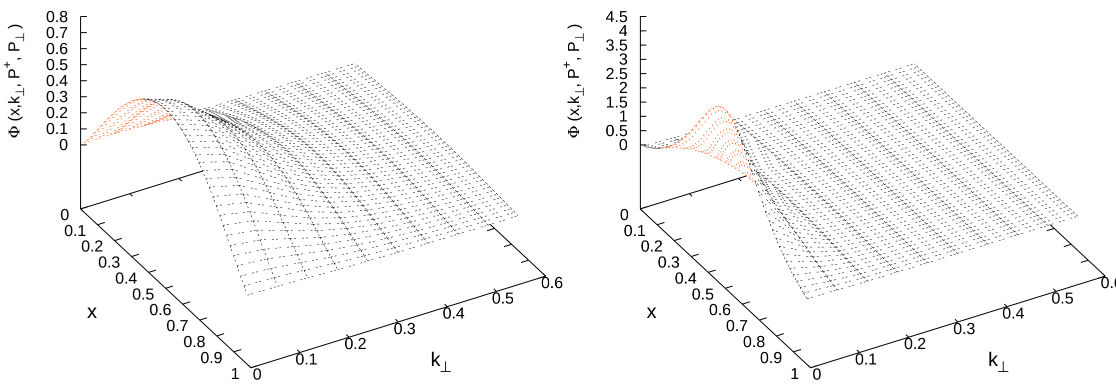

Fig. 2. Pion valence wave functions in vacuum ($\rho = 0$) [left panel] and in medium ($\rho = \rho_0$) [right panel] v.s. x and $k_\perp = |\vec{k}_\perp|$, where $P^+ = m_\pi = m_\pi^*$ and $P_\perp = |\vec{P}_\perp| = 0$. The wave functions are given in the units, $10^{-8}\times(\mathrm{GeV})^{-1}$. Notice that the differences in the vertical axis scales for the left and right panels.

provides information on the nonperturbative regime of the bound state nature of pion due to the quark and antiquark at higher momentum transfer, and it was calculated with different approaches, such as QCD sum rules [35,36], and lattice QCD [37]. Our study here is based on the light-front constituent quark model.

The pion valence wave function in vacuum is normalized by [38,39] (aside from the factor $\sqrt{2}$ difference):

$$\int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3}\Phi_\pi(x,\vec{k}_\perp) \;=\; \frac{f_\pi}{2\sqrt{6}}. \tag{7}$$

This is an important constraint on the normalization of the $q\bar{q}$ wave function [38,39], associated with a probability of finding a pure $q\bar{q}$ state in the pion. According to this normalization, the in-medium pion valence wave function is normalized by replacing $f_\pi \to f_\pi^*$ in the above. Since the pion decay constant in nuclear medium is modified, the pion valence wave function in nuclear medium is also modified via this normalization.

In order to examine more in detail as to how the change in f_π^* impacts on the in-medium pion valence wave function, we show in Fig. 2 the pion valence wave functions in vacuum (left panel) and $\rho = \rho_0$ (right panel).

One can notice that the in-medium pion valence wave function in momentum space has a sharper peak and localized narrower regions both in x and $k_\perp$ than those in vacuum. Of course, the total volume, the quantity integrated over x

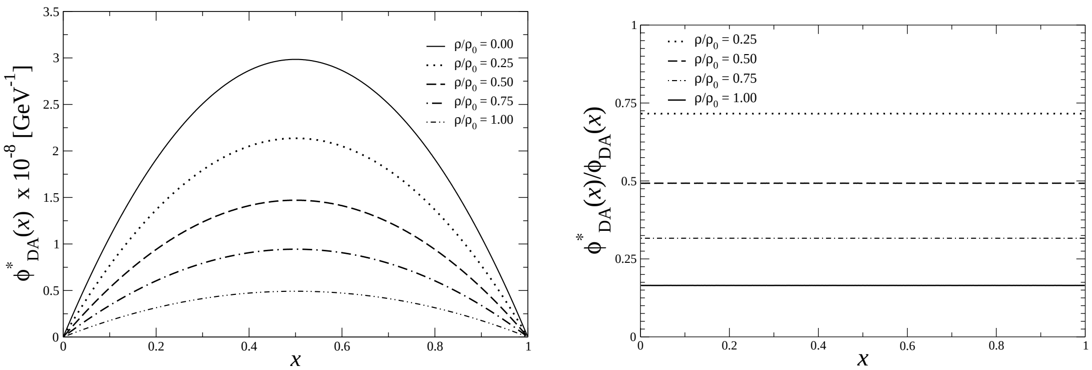

Fig. 3. Pion valence distribution amplitudes (left panel), and the ratios divided by that of the vacuum (right panel). (See also table. 1.)

and $\vec{k}_\perp$, is reduced in medium, corresponding to the reduced f^*_π. This fact is reflected in the wave function in coordinate space, that it becomes spread wider, and generally reduces its hight.

The corresponding pion distribution amplitude (PDA) in medium, $\phi^*_{DA}(x)$ (not normalized to unity), is obtained by integrating over x:

$$\phi^*_{DA}(x) = \int \frac{d^2k_\perp}{16\pi^3}\Phi^*_\pi(x,\vec{k}_\perp). \tag{8}$$

Note that, Eq. (8) holds also for the other pseudoscalar mesons M_{ps} such as kaon and D-meson, by replacing $\Phi^*_\pi(x,\vec{k}_\perp) \to \Phi^*_{M_{ps}}(x,\vec{k}_\perp)$ in the above.

We show in Fig. 3 the PDAs in vacuum $\phi_{DA}(x)$ and in medium $\phi^*_{DA}(x)$ (left panel), and the ratios divided by the vacuum one $\phi_{DA}(x)$ (right panel).

Indeed, the significant reduction of the in-medium PDA ($\phi^*_{DA}(x)$) is obvious in Fig. 3, reflecting the reduction of f^*_π.

Next, we study PDA normalized to unity, or parton distribution function (PDF). By this, we can study the change in shape due to the medium effects. We show in Fig. 4 the calculated PDFs both in vacuum $\phi(x)$ and in medium $\phi^*(x)$ (left panel), and their magnifications (right panel).

The in-medium change in shape is moderate when the nuclear matter densities are small, but it becomes evident when the density is ρ_0.

Next, it may be useful to define *effective parton distribution function* (EFF-PDF) for pion using the valence probability in vacuum η and and in medium η^*. (See Eq. (5) and table 1.) The pion states in vacuum, $|\pi>$, and in medium, $|\pi>^*$, can respectively be written as,

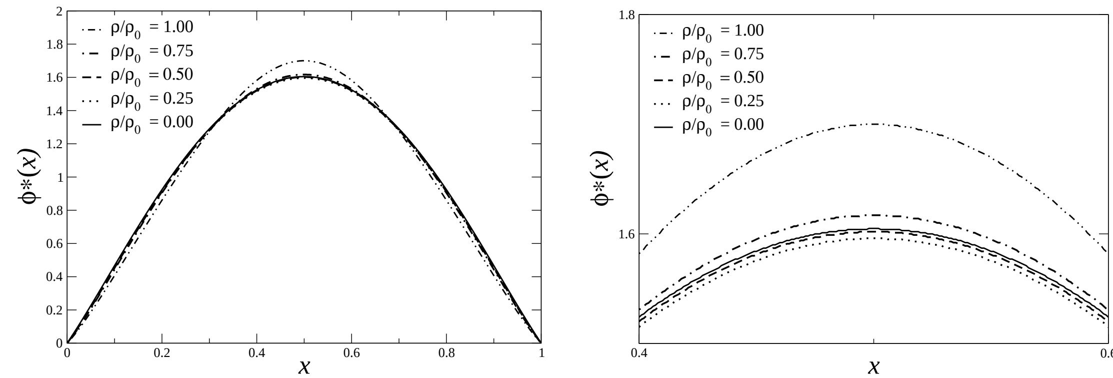

Fig. 4. Pion valence parton distribution functions (left panel), and the maginified

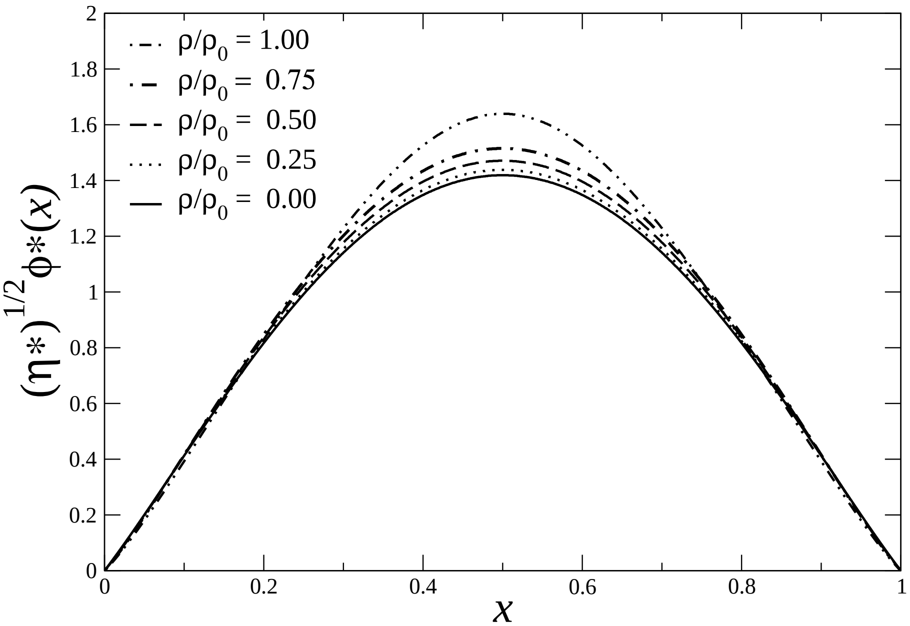

Fig. 5. Effective pion valence parton distribution functions in vacuum and in medium, respectively multiplied by $\sqrt{\eta}$ and $\sqrt{\eta^*}$.

$$|\pi> = \sqrt{\eta}|q\bar{q}> + a|q\bar{q}q\bar{q}> + b|q\bar{q}g> + \cdots \,, \tag{9}$$

$$|\pi>^* = \sqrt{\eta^*}|q\bar{q}>^* + c|q\bar{q}q\bar{q}>^* + d|q\bar{q}g>^* + \cdots \,, \tag{10}$$

where a, b, c and d are constants, and g denotes a gluon, and $+\cdots$ stands for the higher Fock components in the pion states. The quantity η^* in table 1 indicates that the valence $q\bar{q}$ component in the pion state increases in medium as nuclear density increases. The EFFPDFs in vacuum, $\sqrt{\eta}\phi(x)$, and that in medium, $\sqrt{\eta^*}\phi(x)^*$, are shown in Fig. 5. They respectively correspond to the first terms of Eqs. (9) and (10).

Since η^*/η is enhanced in medium, EFFPDF in medium is also enhanced, on the top of the corresponding medium-modified PDF. The obvious enhancement of EFFPDF in medium can be seen around $x = 0.5$. This quantity may be useful when one studies some reactions in medium (in a nucleus) involving a pion, based on a constituent quark picture of pion.

To summarize, we have studied the impact of in-medium effects on the pion valence distribution amplitudes and parton distribution function using a light front constituent quark model, combined with the in-medium input for the constituent quark properties calculated by the quark-meson coupling model. The in-medium constituent quark properties inside the pion are consistently constrained by the saturation properties of symmetric nuclear matter.

The in-medium pion mass is assumed to be the same as that in vacuum, based on the extracted information from the pionic-atom experiment, and some theoretical studies. This information extracted is valid up to around the normal nuclear matter density. Thus, the results obtained in this study, combined with the light-front constituent quark model, are valid up to around the normal nuclear matter density, but cannot discuss reliably the chiral limit, the vanishing limit of the (effective constituent) quark mass. We need to rely on more advanced models of pion to be able to discuss the chiral limit in medium, as well as in vacuum.

Due to the reduction in the pion decay constant in medium, the pion distribution amplitude in medium normalized with the pion decay constant, is appreciably reduced in nuclear medium. Because the valence component probability in medium increases as nuclear density increases, we have defined an effective pion distribution function normalized to the square root of the valence probability in the pion state. This may give some information for the effectiveness of the valence quark picture of pion in nuclear medium. Within the present approach, the effectiveness of the valence quark picture of the pion in medium, becomes more enhanced as nuclear density increases, due to the increase of the valence component in the pion state.

Although the present study is based on a simple, light-front constituent quark model, this is a first step to understand the impact of the medium effects on the internal structure of the pion immersed in nuclear medium. In the future, we plan to make similar studies for kaon, D-meson, and ρ-meson in nuclear medium.

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