

Aerogel Index of Refraction

Meeting 7/26

Calculations

Equations (1): $n = 1.029 \pm .001$

- $\gamma = \alpha - \beta + \arcsin(n(\sin(\beta - \arcsin((\sin\alpha)/n))))$

- $n = \sqrt{\sin^2\alpha + \sin^2(\gamma + \alpha)}$

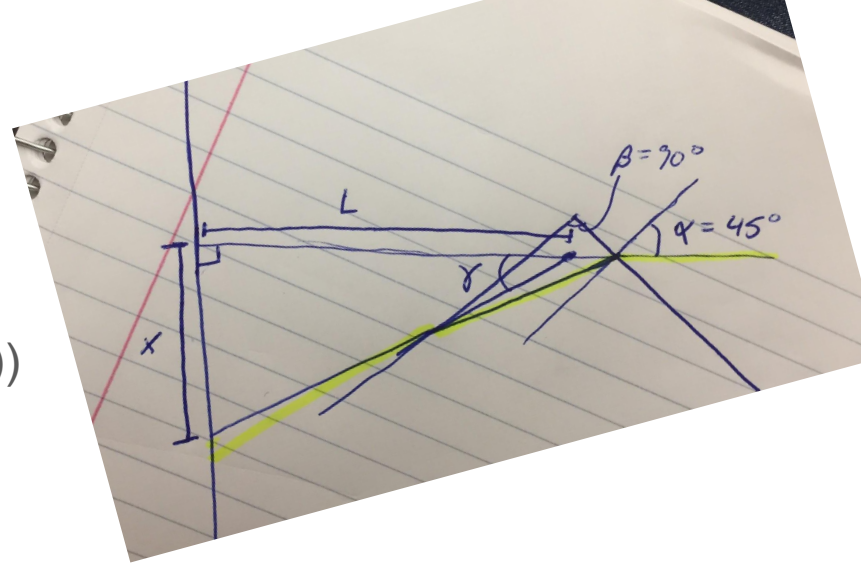
- - assumes β is a right angle

- $\delta n(\alpha, \gamma) = \sqrt{((\partial n / \partial \alpha) \delta \alpha)^2 + ((\partial n / \partial \gamma) \delta \gamma)^2}$

- $\partial n / \partial \alpha = (\cos(\gamma - \alpha) \sin(\gamma - \alpha) + \sin(\alpha) \cos(\alpha)) / (\sqrt{\cos^2(\gamma - \alpha) + \sin^2(\alpha)})$

- $\partial n / \partial \gamma = (-\cos(\gamma - \alpha) \sin(\gamma - \alpha)) / (\sqrt{\cos^2(\gamma - \alpha) + \sin^2(\alpha)})$

- https://docs.google.com/spreadsheets/d/1RAnGFBmL_D1iQxKTcW4YfsI09JV-1IP1-hGFo8ZTCdM/edit#gid=0



Modified Equations (including Beta)

- Looked back at original derivation for Beta = 90 deg. equation to find an expression for n from the original equation
- Found derivatives using W.A. then went back and double checked by hand

$$\begin{aligned} \rightarrow \gamma - \alpha + \beta &= \arcsin(n \sin(\beta - \arcsin(\frac{\sin \alpha}{n}))) \\ \rightarrow \sin(\gamma - \alpha + \beta) &= n \sin(\beta - \arcsin(\frac{\sin \alpha}{n})) \\ &\quad (\text{Use sine subtraction}) \\ \rightarrow \sin(\gamma - \alpha + \beta) &= n \left(\sin(\beta) \cos(\arcsin(\frac{\sin \alpha}{n})) \right. \\ &\quad \left. - \cos(\beta) \sin(\arcsin(\frac{\sin \alpha}{n})) \right) \\ &= n \left(\sin(\beta) \sqrt{1 - \sin^2(\arcsin(\frac{\sin \alpha}{n}))} \right. \\ &\quad \left. - n \cos(\beta) \left(\frac{\sin \alpha}{n} \right) \right) \\ &\quad \begin{array}{l} \uparrow \text{Use } \sin^2 + \cos^2 = 1 \\ \uparrow \text{Use } \sin(\arcsin(x)) = x \end{array} \\ \rightarrow \sin(\gamma - \alpha + \beta) &= n \left(\sin(\beta) \sqrt{1 - \frac{\sin^2 \alpha}{n^2}} \right. \\ &\quad \left. - \cos(\beta) (\sin \alpha) \right) \\ \rightarrow \sin(\gamma - \alpha + \beta) + \sin(\alpha) \cos(\beta) &= n \sin \beta \sqrt{1 - \frac{\sin^2 \alpha}{n^2}} \end{aligned}$$

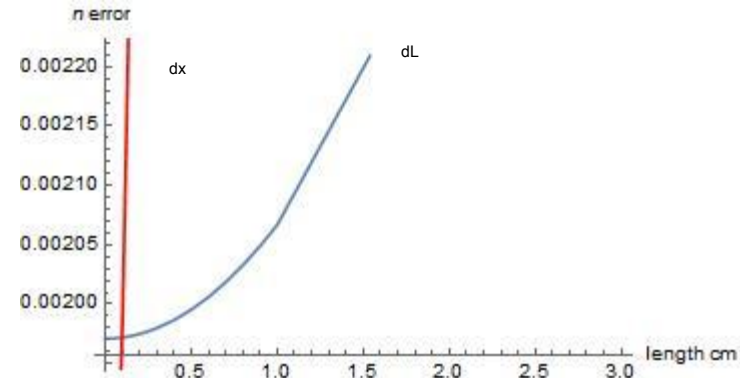
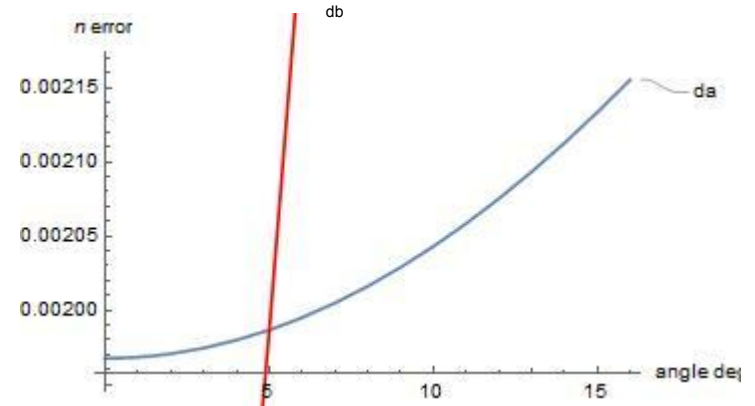
$$\begin{aligned} \rightarrow \sin(\gamma - \alpha + \beta) + \sin(\alpha) \cos(\beta) &= \frac{n \sin(\beta) \sqrt{n^2 - \sin^2 \alpha}}{n} \\ \rightarrow \frac{\sin(\gamma - \alpha + \beta) + \sin(\alpha) \cos(\beta)}{\sin(\beta)} &= \sqrt{n^2 - \sin^2 \alpha} \\ \rightarrow n^2 &= \left(\frac{\sin(\gamma - \alpha + \beta) + \sin(\alpha) \cos(\beta)}{\sin(\beta)} \right)^2 \\ &\quad + \sin^2 \alpha \\ \rightarrow n &= \left(\frac{\sin(\gamma - \alpha + \beta) + \sin(\alpha) \cos(\beta)}{\sin(\beta)} \right)^2 \\ &\quad + \sin^2 \alpha \end{aligned}$$

☆☆☆
(expression for n from 1st equation)

Error Analysis

- The Original Equation is very sensitive to error in Beta and x, not as sensitive to error in Alpha and L
- An error of less more than 2 deg. In Beta means an error of abt. 0.002 in n
- The x measurements are already precise, just need to double check to make sure everything is aligned properly
- Used Starting measurements of:

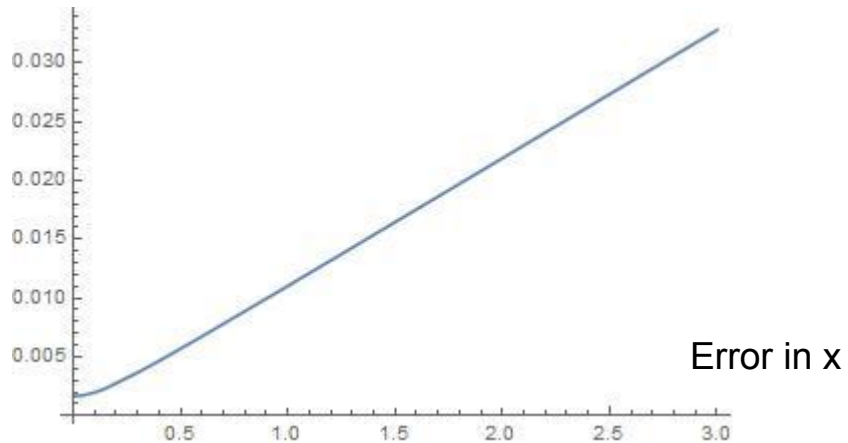
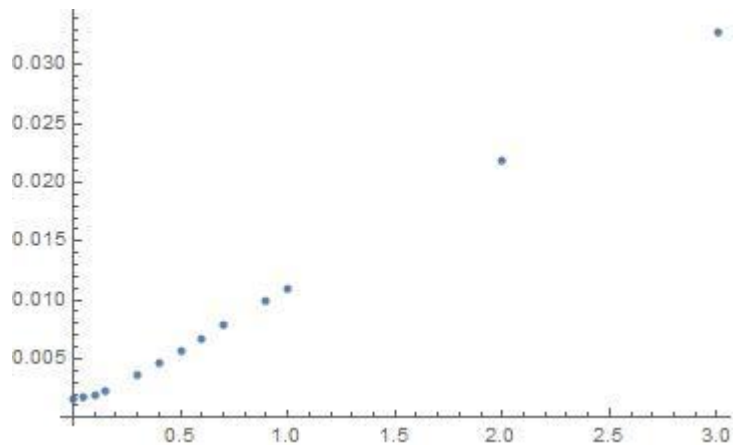
$A_i = 45 \text{ deg}$	$dA_i = 5 \text{ deg}$
$B_i = 90 \text{ deg}$	$dB_i = 5 \text{ deg}$
$L_i = 45.6 \text{ cm}$	$dx_i = .1 \text{ cm}$
$x_i = 2.6 \text{ cm}$	$dL_i = .4 \text{ cm}$



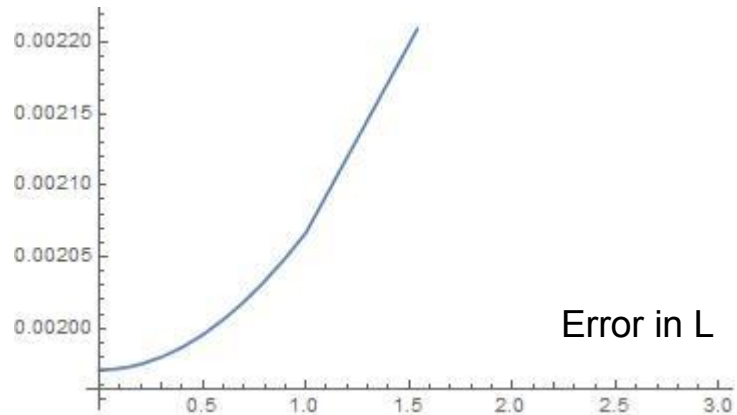
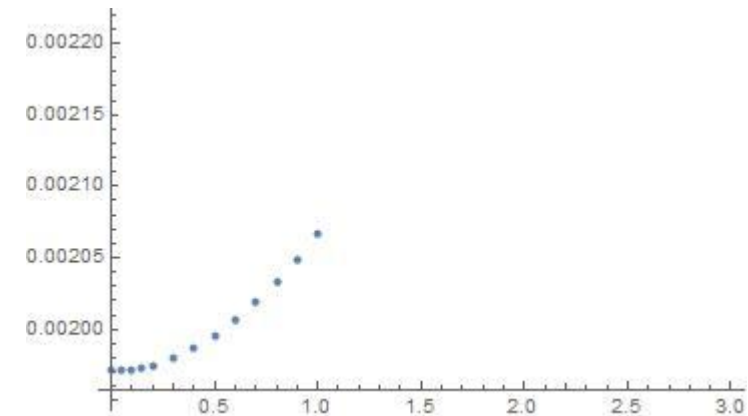
<https://docs.google.com/spreadsheets/d/1HJZ5eK6huzZO94GtAjLsAfnvpXzxYXvuAw74P8qCl84/edit#gid=0>

Error in x and L

Error
in n

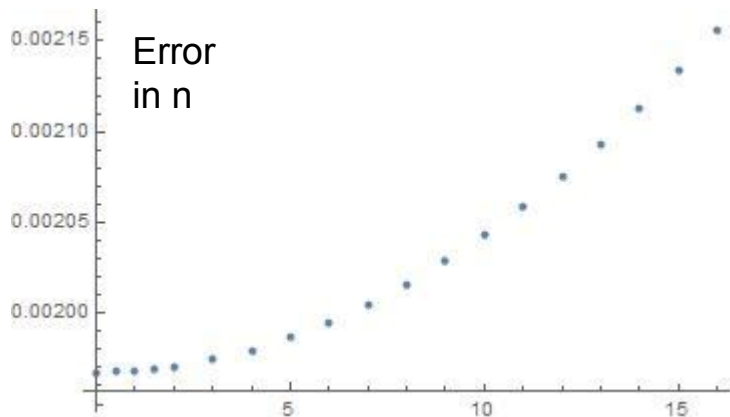


Error in n

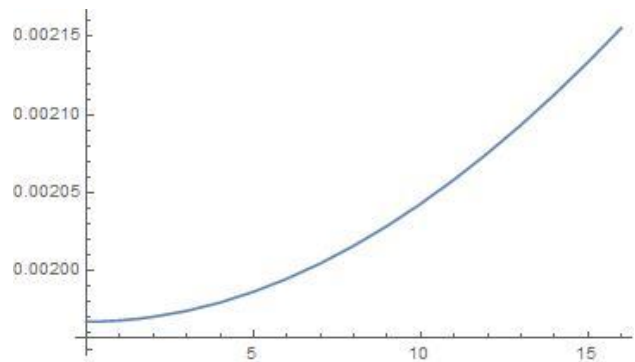
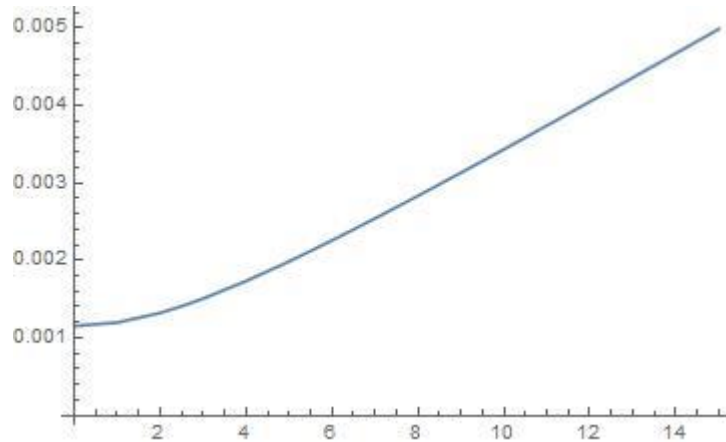


Error in L

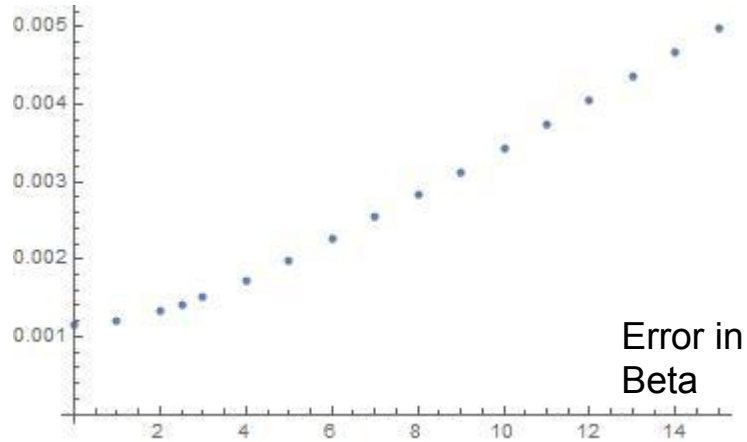
Error in Alpha and Beta



Error in n



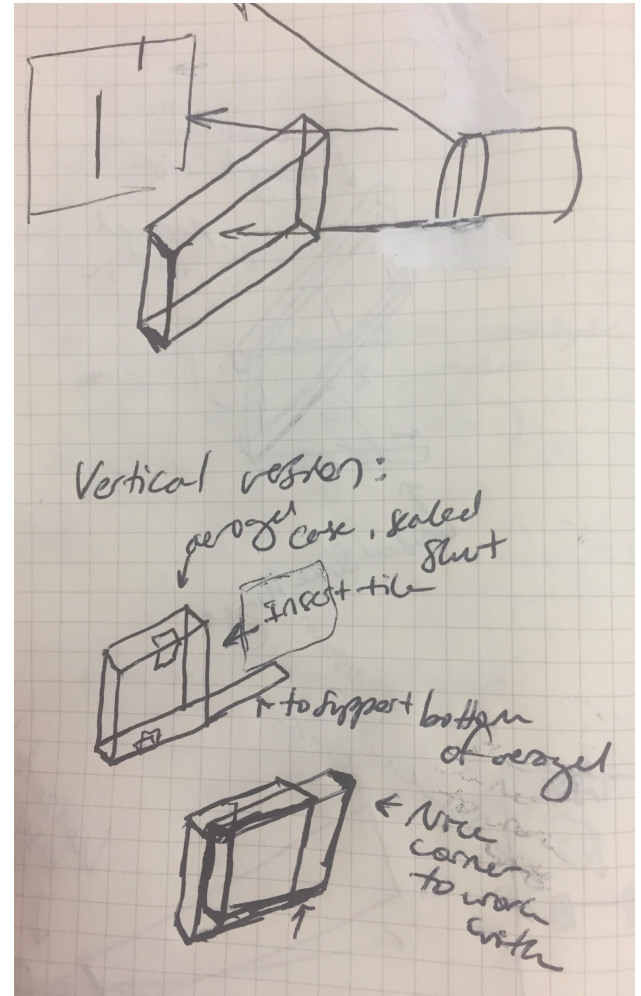
Error in Alpha



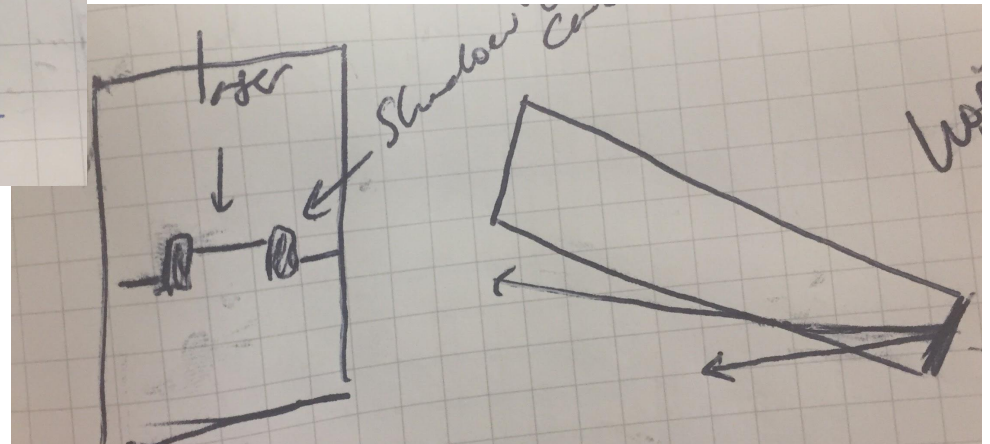
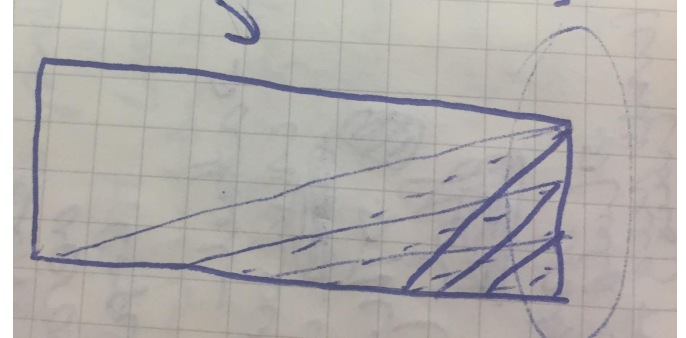
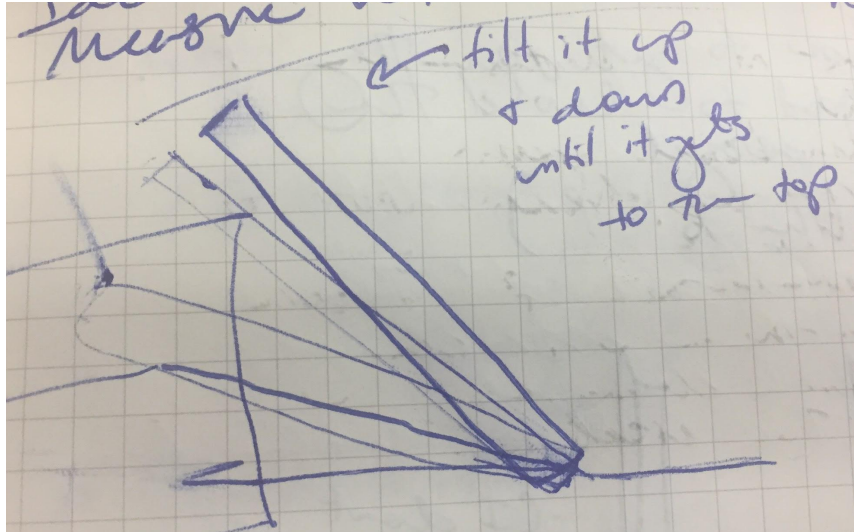
Error in Beta

Ideas for Measuring n across the long edges

- Change the angle and depth of the laser position (using graph paper)

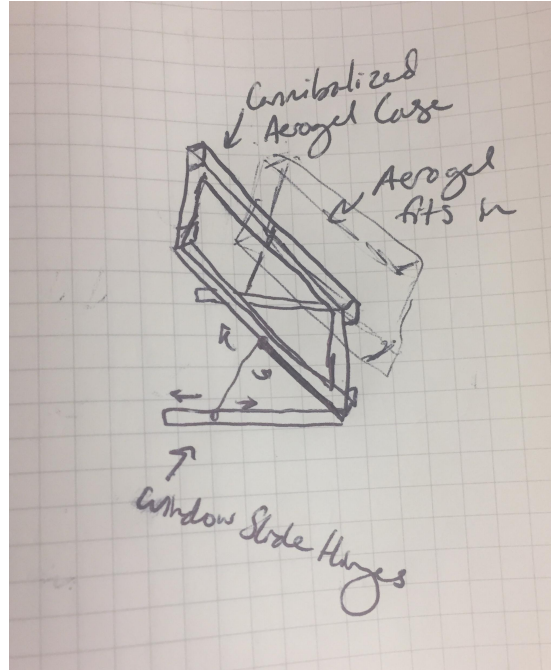


Idea for Measuring n across the long edges

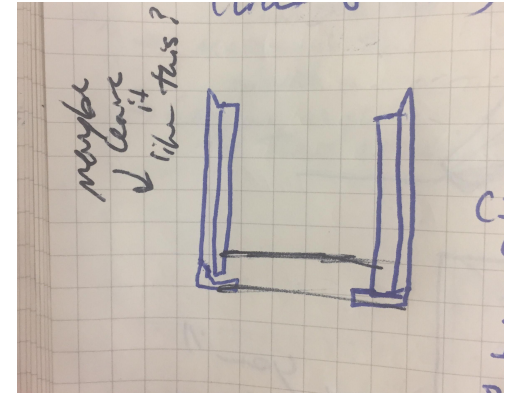


Idea for Measuring n across the long edges

- Preferable to just vertical version because you can use a greater portion of the tile
- Also doesn't put all the weight of the tile on one edge (safer?)
- Tilting movement - Aerogel moved instead of laser



Might be hard to measure angles - definitely overly complicated anyway



https://www.amazon.com/Truth-Hardware-Cement-Hinge-28-15-5/dp/B005E0MXLK?ref=fscplp_pl_dp_15

Transmittance Measurements

- Take a vertical measurement of n near the edge of the tile and then use transmittance measurements to n across the rest?
- Fresnel equations
 - $R_s = (n_1 \cos(\theta_i) - n_2 \sqrt{1 - (n_1/n_2) \sin^2(\theta_i)}) / (n_1 \cos(\theta_i) + n_2 \sqrt{1 - (n_1/n_2) \sin^2(\theta_i)})$
 - $R_p = (n_1 \sqrt{1 - (n_1/n_2) \sin^2(\theta_i)} - n_2 \cos(\theta_i)) / (n_1 \sqrt{1 - (n_1/n_2) \sin^2(\theta_i)} + n_2 \cos(\theta_i))$
 - $R_{\text{eff}} = \frac{1}{2} (R_s + R_p)$
- $T = (1-R)^2 + \alpha \cdot \text{thickness}$
 - (R = Reflectance and α = absorbance)

Next Week

- Examine effect of error more closely
- Retake measurements with more accurate x and β
- Vertical measurements? Transmittance?
- Model of index variations in the tile
- Meeting with Nick - Mathematica