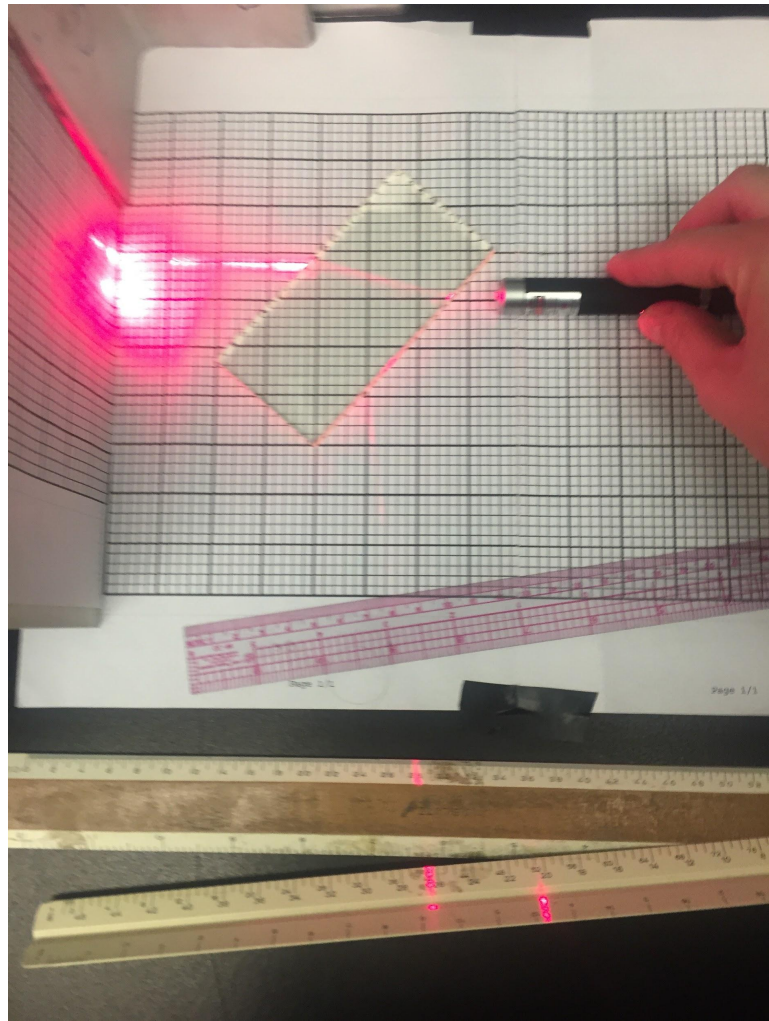
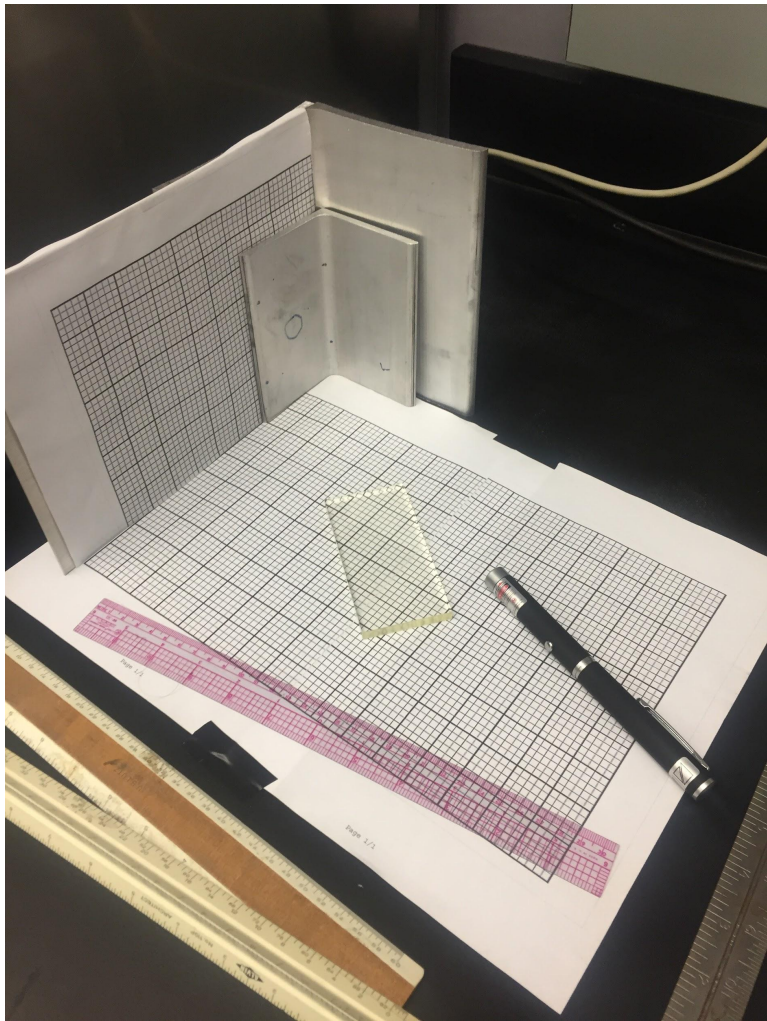
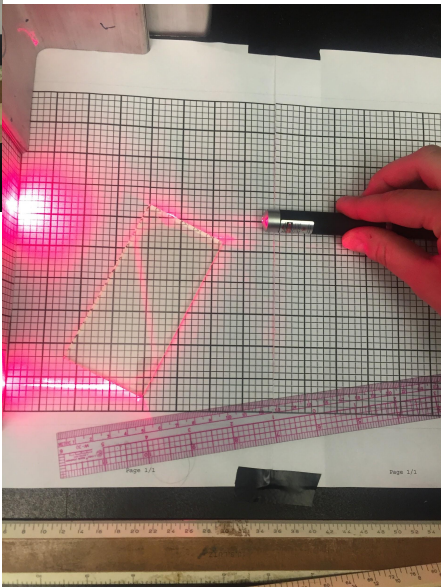
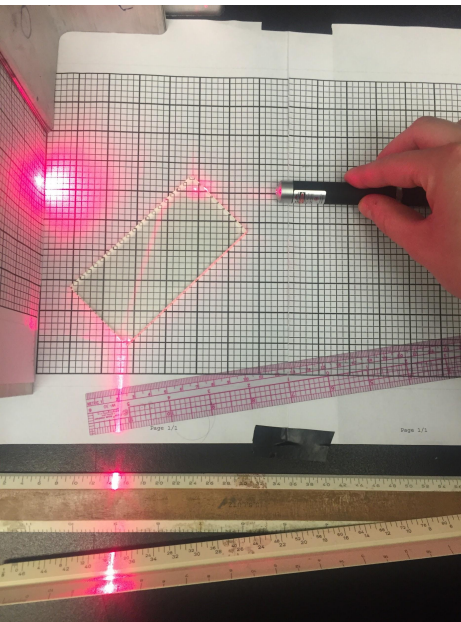
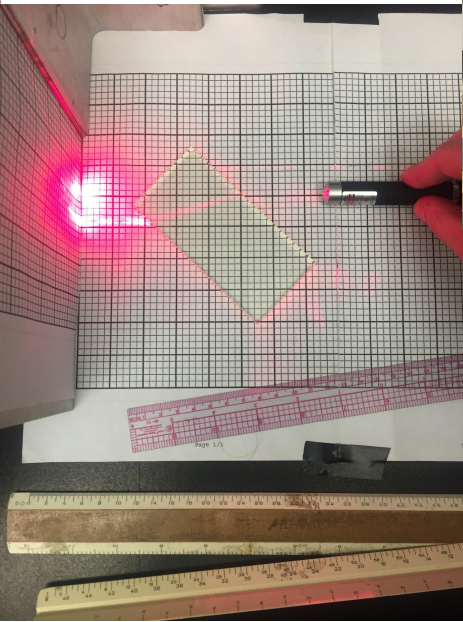
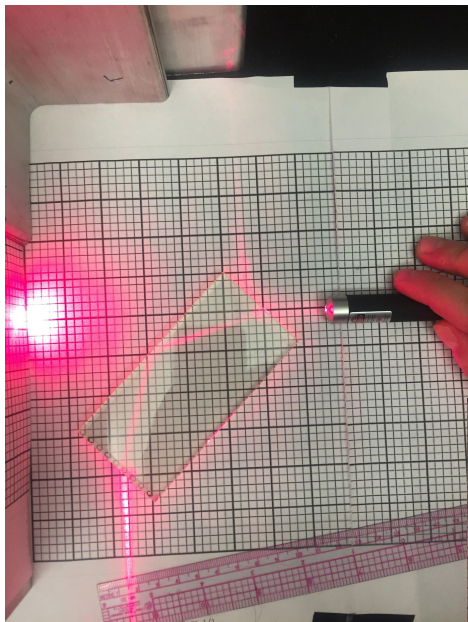


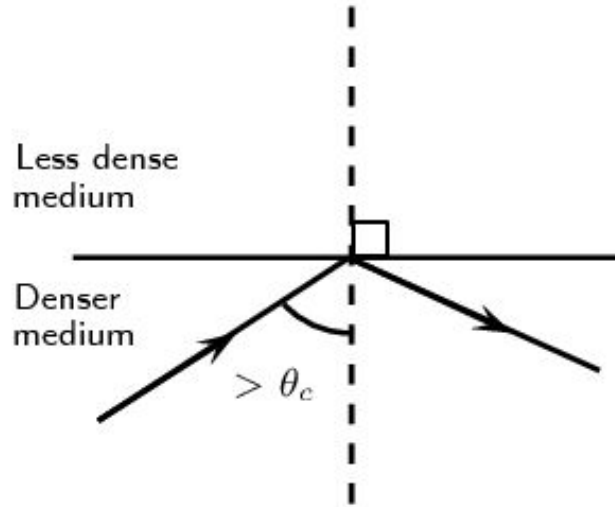
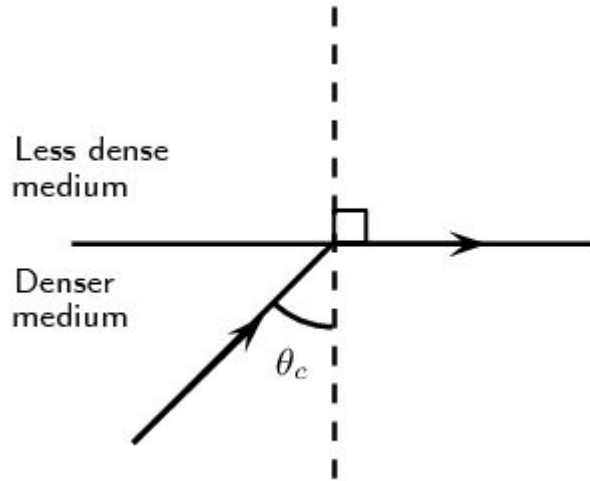
Aerogel Index of Refraction

Meeting Updates





The Critical Angle



Index of Refraction (1)

$$\gamma = \alpha - \beta + \arcsin(n(\sin(\beta - \arcsin(\frac{\sin \alpha}{n}))))$$

$$n = \sqrt{\sin^2 \alpha + \sin^2(\gamma + \alpha)}$$

$\rightarrow \beta + d = 180^\circ \rightarrow d = 180^\circ - \beta$
 $\rightarrow d = 180^\circ - (a + b)$
 $\rightarrow \beta = a + b$
 $\rightarrow \gamma = \theta_1 + \theta_2$
 $\rightarrow \theta_1 = d - a$
 $\rightarrow \theta_2 = c - b$
 $\rightarrow \gamma = d - a + c - b$
 $\rightarrow \gamma = d - (a + b) + c$
 $\rightarrow \gamma = d - \beta + c$

β not necessarily 90° , not necessarily $= d$

$\rightarrow \frac{\sin \alpha}{\sin a} = \frac{n \sin c}{n \sin b}$
 $\Rightarrow \frac{\sin \alpha}{\sin a} = \frac{\sin c}{\sin b}$
 $\rightarrow \frac{\sin c}{\sin b} = \frac{n \sin c}{n \sin b}$
 \rightarrow Let $\frac{n \sin c}{n \sin b} = n$
 $\rightarrow c = \arcsin(n \sin(\beta - \arcsin(\frac{\sin \alpha}{n})))$
 $\rightarrow \sin c = n \sin(\beta - \arcsin(\frac{\sin \alpha}{n}))$
 $\rightarrow \sin b = \sin(\beta - \arcsin(\frac{\sin \alpha}{n}))$
 $\rightarrow \sin a = \frac{\sin \alpha}{n}$
 $\rightarrow a = \arcsin(\frac{\sin \alpha}{n})$

$$\rightarrow d = \gamma + c$$

$$\rightarrow \gamma = d - c$$

$$\rightarrow d - c + b = \gamma$$

$$\rightarrow \gamma = d + b - c$$

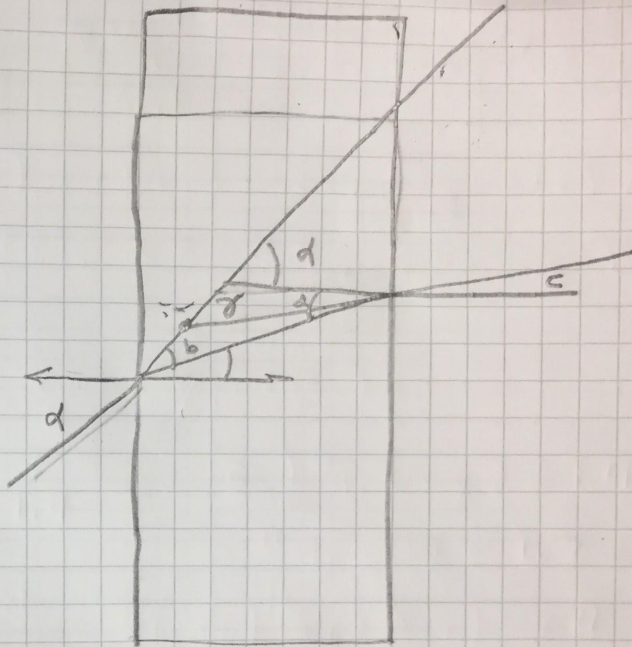
$$\rightarrow d = b + c$$

$$\rightarrow \frac{\delta n d}{\delta n (d-b)} = n = \frac{\delta n c}{\delta n d}$$

$$\rightarrow \frac{\delta n d}{\delta n c} = \frac{\delta n (d-b)}{\delta n d} = \frac{\delta n (b+d-b)}{\delta n d} = 1$$

$$\rightarrow d = c \quad \therefore \gamma = 0$$

\therefore Why previous equation

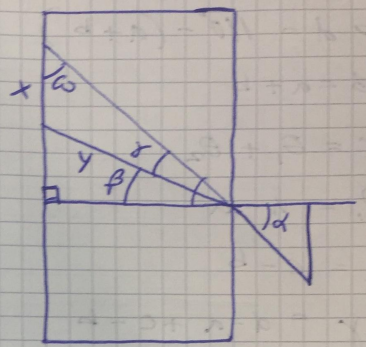


The index cannot be calculated using the first equation when the laser is shone through the body of the prism rather than across the angle

Index of Refraction (2)

- $n = (\sin \alpha) / (\sin(\alpha - \arcsin(x \cos \alpha) / y))$
- $n = (\sin \alpha) / (\sin \beta)$

$\rightarrow \beta = \alpha - \gamma$
 $\rightarrow \omega = 180 - \alpha - \beta$
 $\rightarrow \omega = 90 - \alpha$



$\rightarrow \frac{y}{\sin \omega} = \frac{x}{\sin \gamma}$
 $\rightarrow \gamma = \arcsin\left(\frac{(\sin \omega) x}{y}\right)$
 $\rightarrow \beta = \alpha - \arcsin\left(\frac{(\sin \omega) x}{y}\right)$
 $\rightarrow \frac{\sin \alpha}{\sin \beta} = \frac{n_{air}}{n_{prism}} = n$

$\rightarrow n = \frac{\sin \alpha}{\sin\left(\alpha - \arcsin\left(\frac{\sin(90 - \alpha) x}{y}\right)\right)}$

$\rightarrow n = \frac{\sin \alpha}{\sin\left(\alpha - \arcsin\left(\frac{x \cos(\alpha)}{y}\right)\right)}$

Measurements and Equations (2)

https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK_qlBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1868993415

- $n = (\sin \alpha) / (\sin(\alpha - \arcsin(x \cos \alpha) / y))$
- $n = (\sin \alpha) / (\sin \beta)$
- $\delta n(\alpha, x, y) = \sqrt{((\partial n / \partial \alpha) \delta \alpha)^2 + ((\partial n / \partial x) \delta x)^2 + ((\partial n / \partial y) \delta y)^2}$
- $\partial n / \partial \alpha =$

Handwritten derivation of $\partial n / \partial \alpha$ on lined paper:

$$\rightarrow = \frac{\cos \alpha (\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y}))) - \sin \alpha (\cos(\alpha - \arcsin(\frac{x \cos \alpha}{y}))) (1 - \frac{x \sin \alpha}{y \sqrt{1 - (\frac{x \cos \alpha}{y})^2}})}{(\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})))^2}$$

Handwritten derivation of $\partial n / \partial y$ on lined paper:

$$\rightarrow \frac{\partial n}{\partial y} = \frac{-\frac{x \cos \alpha}{y^2 (1 - (\frac{x \cos \alpha}{y})^2)^{\frac{3}{2}}} \sin(\alpha - \arcsin(\frac{x \cos \alpha}{y}))}{(\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})))^2}$$

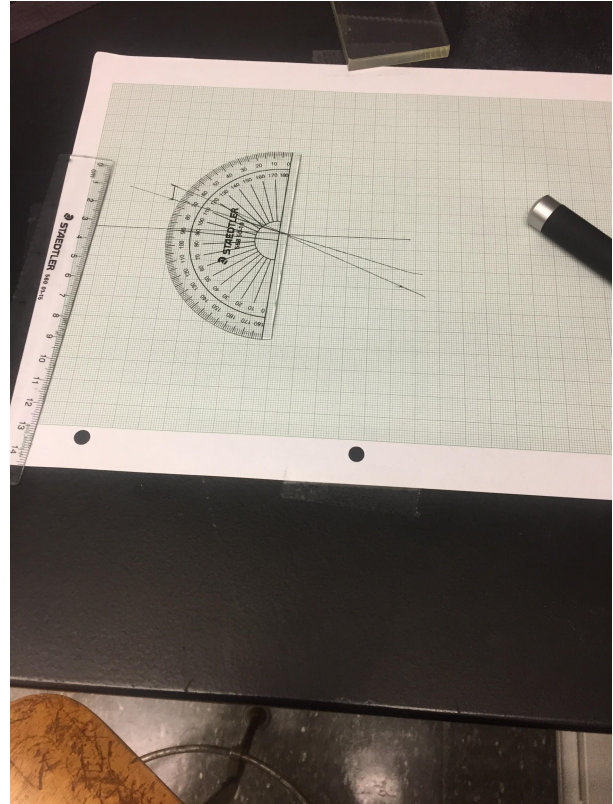
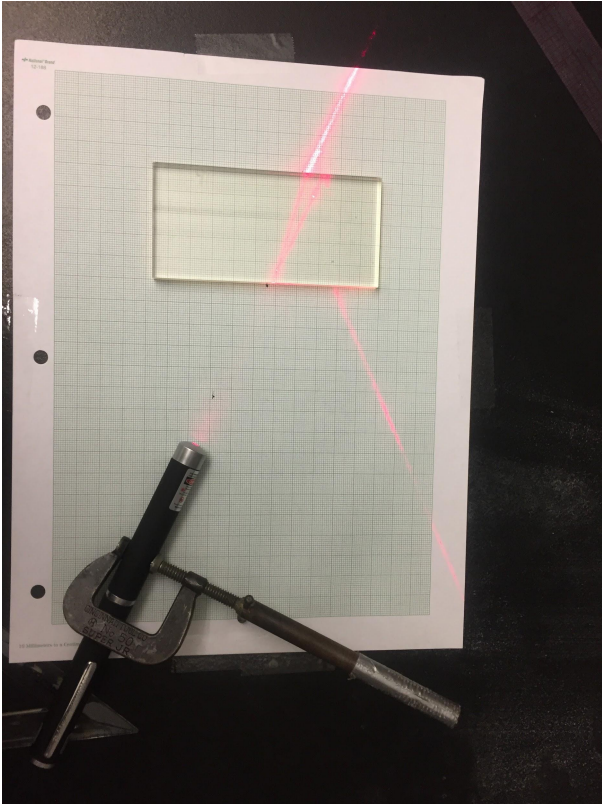
Handwritten derivation of $\partial n / \partial x$ on lined paper:

$$\rightarrow \frac{\partial n}{\partial x} = \frac{-\sin \alpha \cos \alpha}{y (1 - (\frac{x \cos \alpha}{y})^2)^{\frac{3}{2}} (\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})))^2}$$

Original Measurement Approach

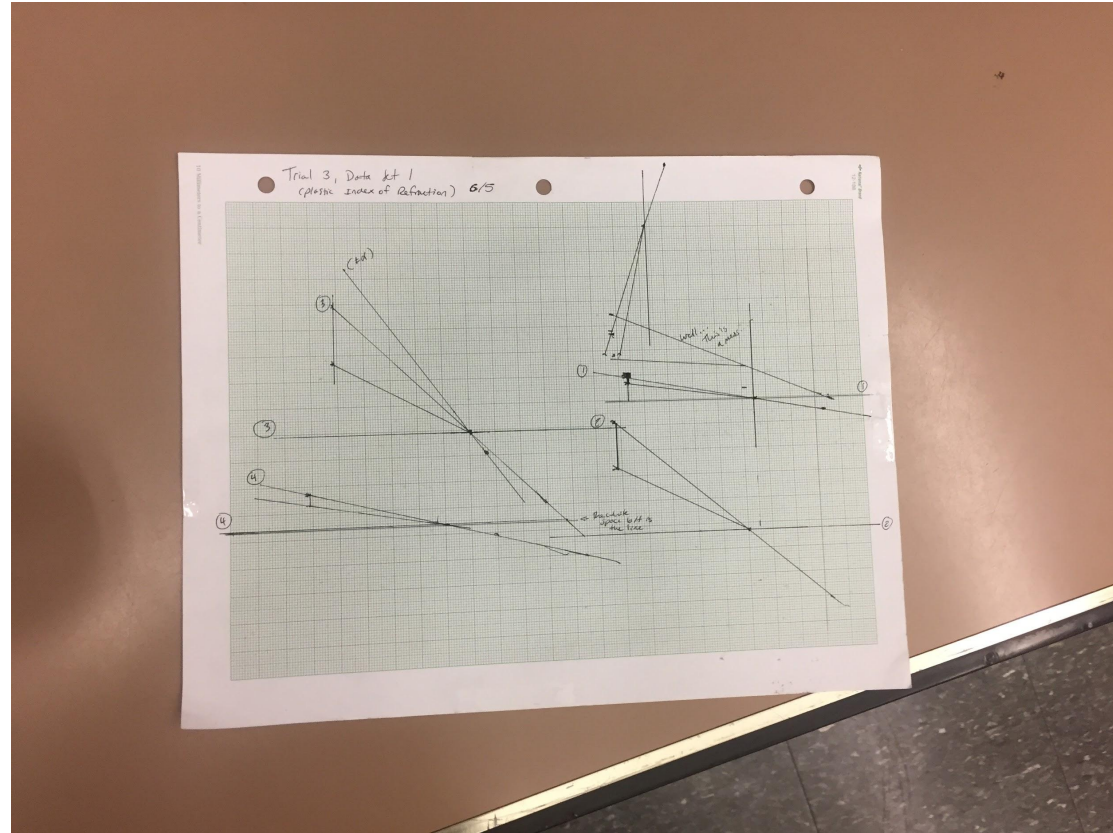
- Measuring while the laser was on
- Using original graph paper as reference
- Taking whole laser width as systematic error with additional random error
- Taking each measurement of 4 values as a set
- Problems:
 - Beam lifted up and put down to be able to see angles - eventually propped it up instead
 - Laser wouldn't stay on
 - Prism and laser easily inadvertently moved
 - Protractor placed on top of prism - additional error from perspective
 - Small protractor, beam covers multiple degrees
 - Beam refracted through transparent ruler and protractor
 - Estimated error of +/- .2 for n

New Approach



New Approach

- Marking the location of the beam and measuring afterwards
- Using more precise graph paper
- Estimating the position of the edge of the laser beam (set 1) and then the center of the laser beam (set 2) as the point of measurement
- Taking each measured value independently
- Problems:
 - Protractor
 - Lines and points contribute to possible systematic error
 - Still error of .06+ for n
 - Data doesn't agree \rightarrow error underestimated



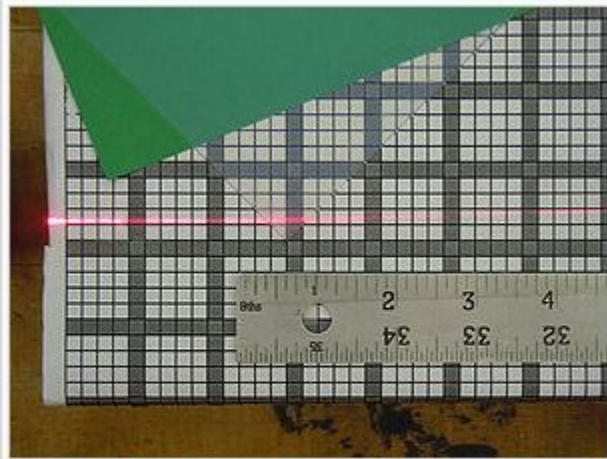
https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK_qlBFUmILhNHlCuCy1psQFxFUU7tCow/edit#gid=1346628582 - Spreadsheet

Possible Image Analysis Software

- ImageJ - Free Image Analysis Software <https://imagej.nih.gov/ij/features.html>
 - “Measure area, mean, standard deviation. Measure lengths and angles. Use real world measurement units such as millimeters. Generate histograms and profile plots”
- Fiji - ImageJ Package <http://fiji.sc/>
 - Same measurement features as ImageJ - Tested this week
- Digimizer - not free <https://www.digimizer.com/features.php>
 - ‘Define unit of measurement, measure distances and lengths of line segments or path, measure angles, locate middle of line segment, measurements on perpendicular lines, fit line’
- Adobe Photoshop - <https://helpx.adobe.com/photoshop/using/measurement.html>
 - “Using the Photoshop Measurement feature you can measure any area defined with the Ruler tool or with a selection tool, including irregular areas selected with the Lasso, Quick Select, or Magic Wand tools. You can also compute the height, width, area, and perimeter, or track measurements of one image or multiple images”

2013 Project

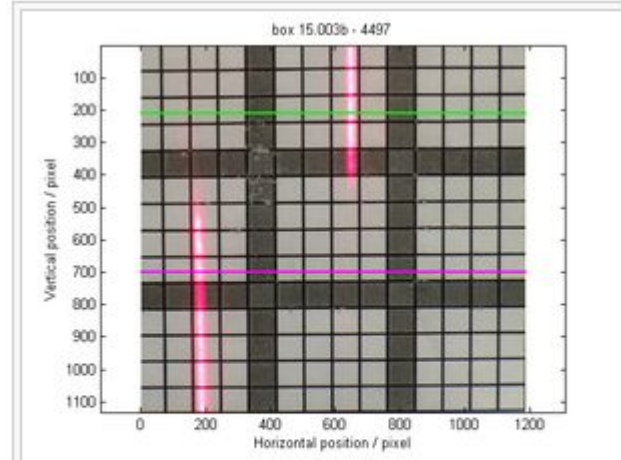
Copying set up from 2013 (Images from http://www.vsl.cua.edu/cua_phy/index.php/MainPage:Nuclear:KaonDetector:AerogelCharacteristics)



2) Part of the laser sheet pass through the aerogel tile, being deflected by its refractive index. The other part pass on the top of the tile, going direct to the reference paper target.

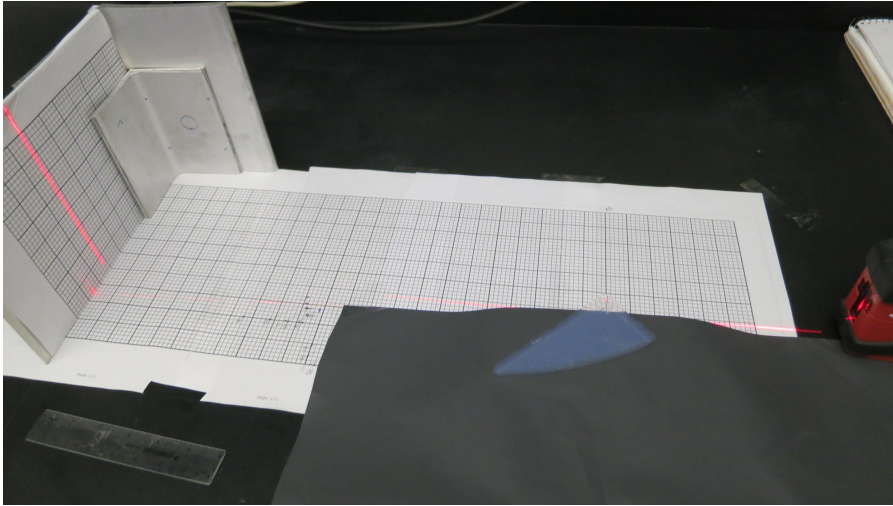


1) Experimental setup for the measurement of the aerogel refractive index. A laser sheet is refracted by the tile and hit a target paper. The deviation of the laser can be measured for the calculation of the refractive index.

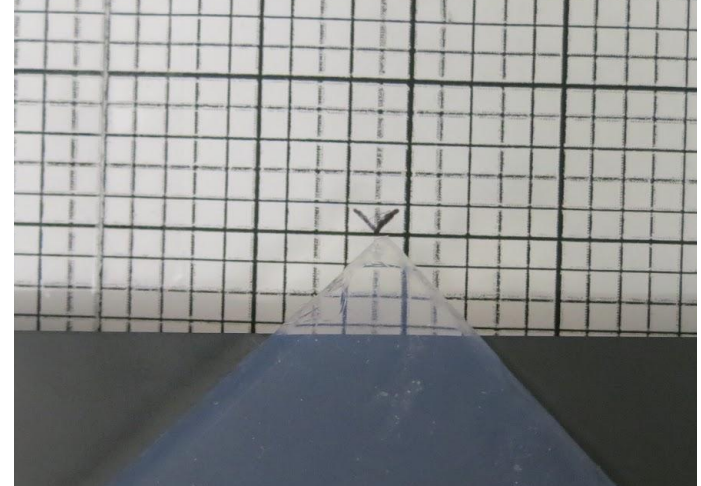
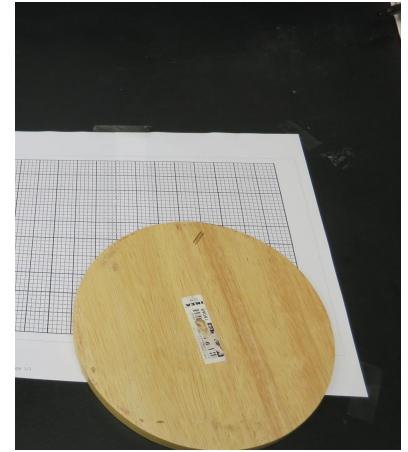


3) Picture of the reference paper with the direct (top) and refracted (bottom) laser beam. Two lines are selected for the pixel intensity analysis, looking for the peaks identification.

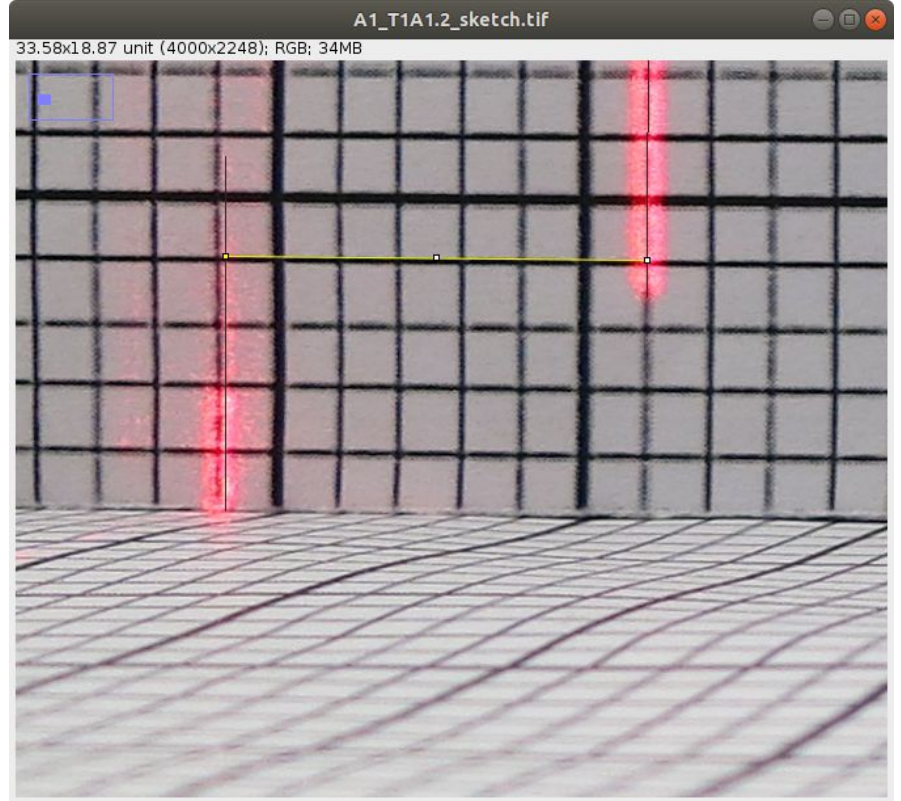
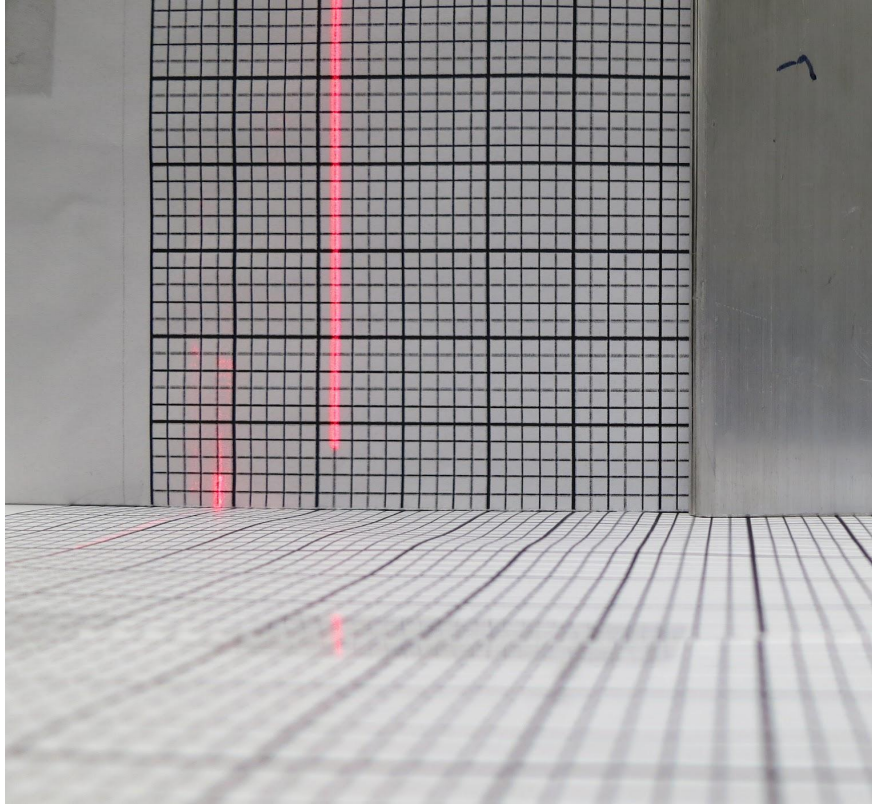
Aerogel Setup



- Removes variables of angle and distance from target
- New variable - distance inside aerogel
- Measure distances on the target using pictures and distance from target by hand



Aerogel Set up and Image Analysis



Calculations

Equations (1): $n = 1.029 \pm .001$

- $\gamma = \alpha - \beta + \arcsin(n(\sin(\beta - \arcsin((\sin\alpha)/n))))$

- $n = \sqrt{\sin^2\alpha + \sin^2(\gamma + \alpha)}$

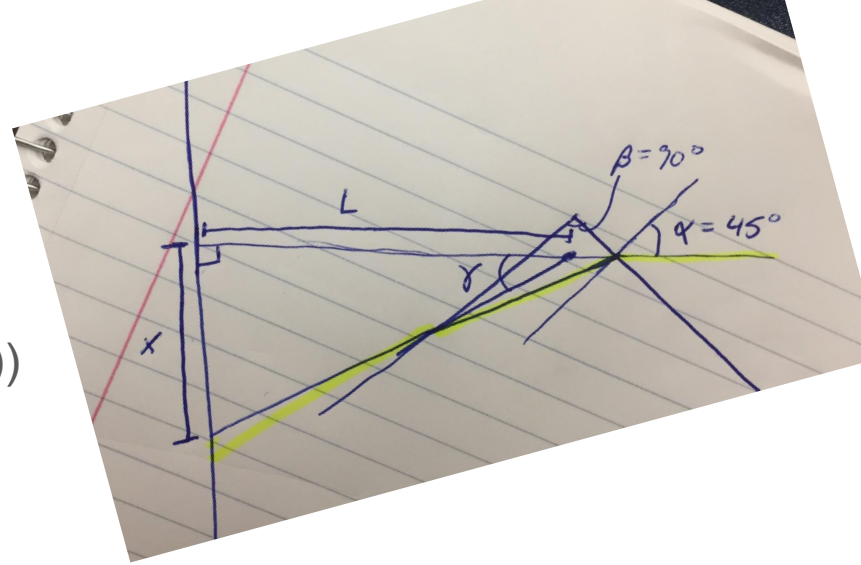
- - assumes β is a right angle

- $\delta n(\alpha, \gamma) = \sqrt{((\partial n / \partial \alpha) \delta \alpha)^2 + ((\partial n / \partial \gamma) \delta \gamma)^2}$

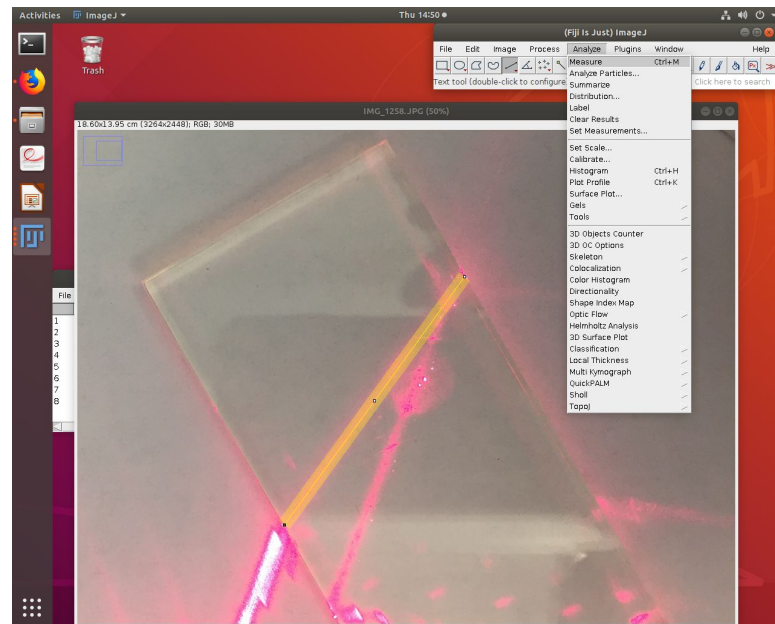
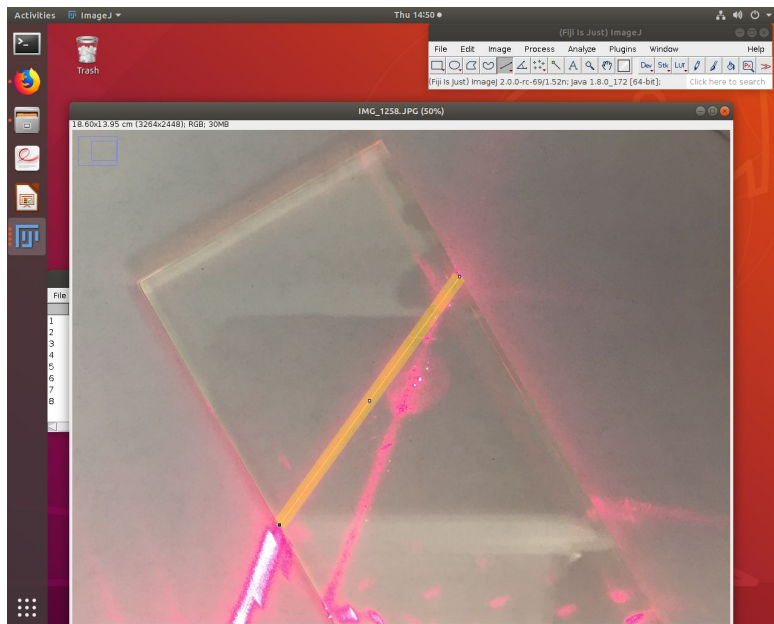
- $\partial n / \partial \alpha = (\cos(\gamma - \alpha) \sin(\gamma - \alpha) + \sin(\alpha) \cos(\alpha)) / (\sqrt{\cos^2(\gamma - \alpha) + \sin^2(\alpha)})$

- $\partial n / \partial \gamma = (-\cos(\gamma - \alpha) \sin(\gamma - \alpha)) / (\sqrt{\cos^2(\gamma - \alpha) + \sin^2(\alpha)})$

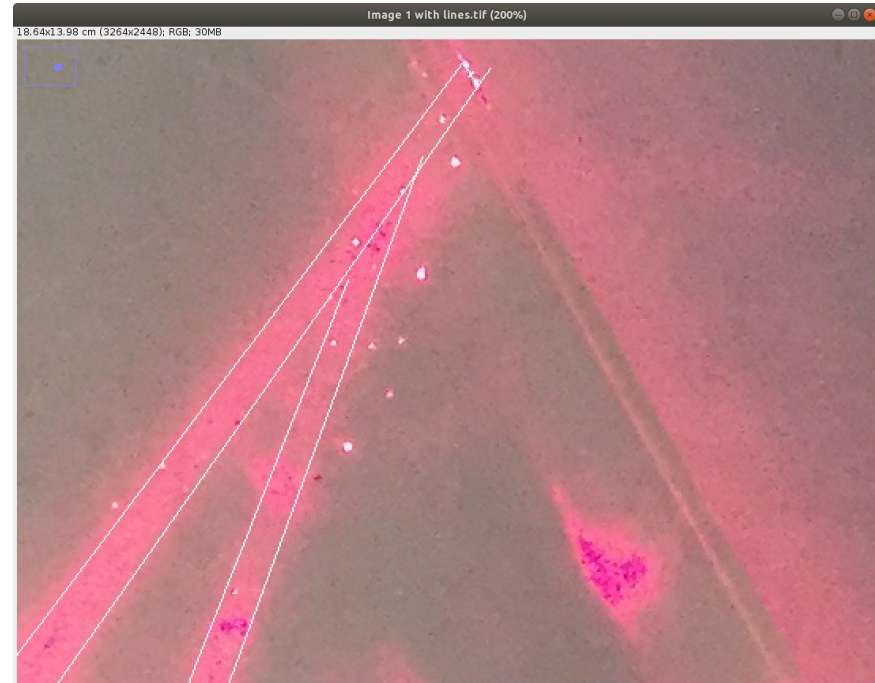
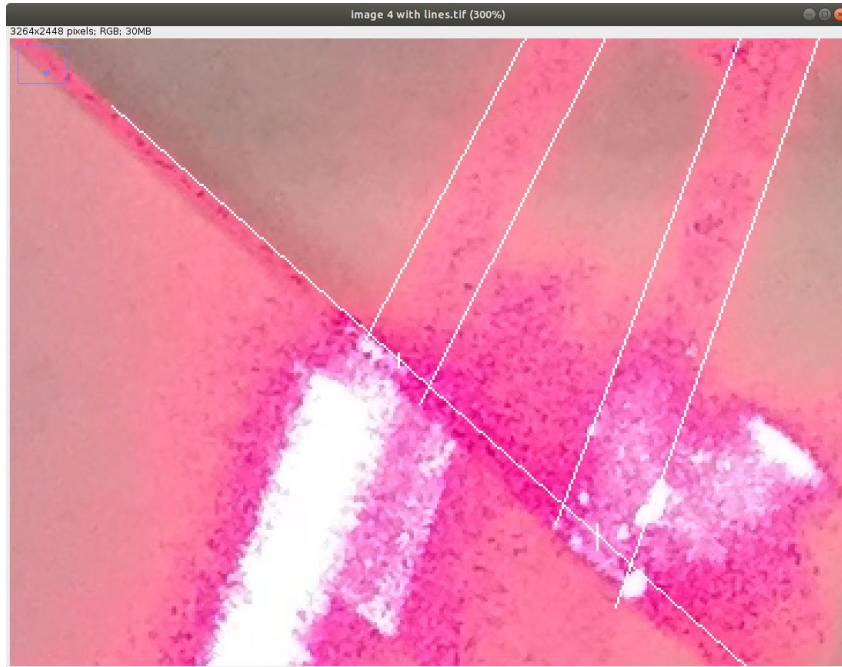
- https://docs.google.com/spreadsheets/d/1RAnGFBmL_D1iQxKTcW4YfsI09JV-1IP1-hGFo8ZTCdM/edit#gid=0



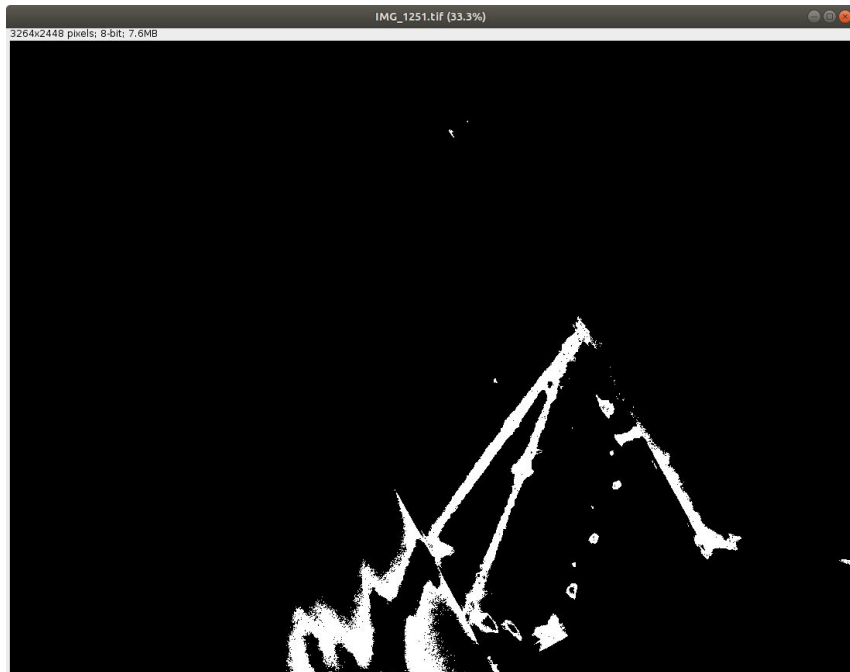
ImageJ



ImageJ



ImageJ



Activities ImageJ Fri 11:30 (Fiji is Just) ImageJ

File Edit Image Process Analyze Plugins Window Help

Fiji is Just! ImageJ 2.0.0-rc-69/1.52n; java 1.8.0_172 [64-bit]; Click here to search

Import to results _ijm

```
File Edit Language Implem Bun Tools Tabs Options
Import to results _ijm
10 macro "Import Results Table" {
11     requires("1.35n");
12     lineseparator = "\n";
13     cellseparator = "\t";
14
15     // copies the whole RT to an array of lines
16     lines=split(File.openAsString(""), lineseparator)
17
18     // recreates the columns headers
19     labels=split(lines(0), cellseparator);
20     label=...

```

Results

File	Edit	Font
X	Y	Angle
1786	1787	0
1787	1787	0
1788	1787	0
1789	1787	0
1790	1787	0
1791	1787	0
1789	1789	0
1790	1789	0
1791	1789	0
1789	1790	0
1790	1790	0
1791	1790	0
1789	1791	0
1790	1791	0
1791	1791	0

Sample x and y for line fit.csv

File	Edit	Font	
X	Y	Angle	Length
550	1786	0	0
569	1781	0	0
570	1780	0	0
576	1789	0	0
576	1790	0	0
576	1791	0	0
577	1789	0	0
577	1790	0	0

Plot of Results

NahMaN (613x355); RGB; 850K

A screenshot of the ImageJ software interface showing a linear fit analysis. The main window displays a binary image of a handwritten digit '4'. A 'Plot of Results' window is open, showing a scatter plot of the digit's pixels with a blue line representing the linear fit. The plot axes are labeled 'X' and 'Y'. The status bar at the top indicates the file is 'IMG_1251.tif' with a zoom level of 33.3%. The status bar at the bottom of the plot window shows 'NahMaN (613x355); RGB; 850K'. A 'Log' window is also open, displaying the results of the linear fit, including the formula $y = a + bx$, the number of iterations (1), the time (7 ms), the sum of residuals squared (92408682.6), the standard deviation (52.62885), the R-squared value (0.97935), and the parameters $a = 3490.46633$ and $b = -2.39246$.

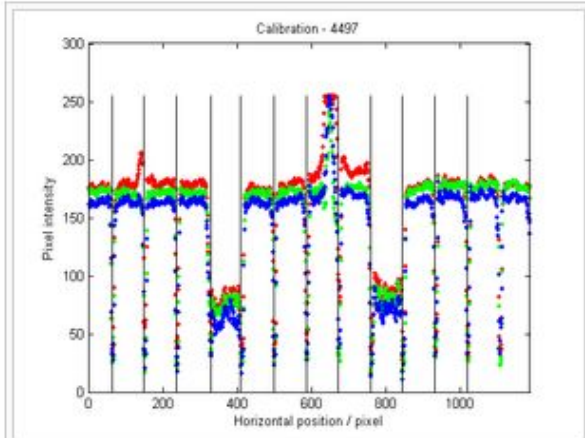
Mathematica

- Working with Nick to come up with image analysis program
- Many of the same tools as imagej
 - Thresholding, edge tracing, Hough and Radon Transform
- Image info (matrix)
- Began with measuring distances of lasers inside plastic prism
 - He suggested a program where someone would select the corners of an image and the laser beams and then the angle and distance would be automatically measured
 - Do we need a streamlined process?

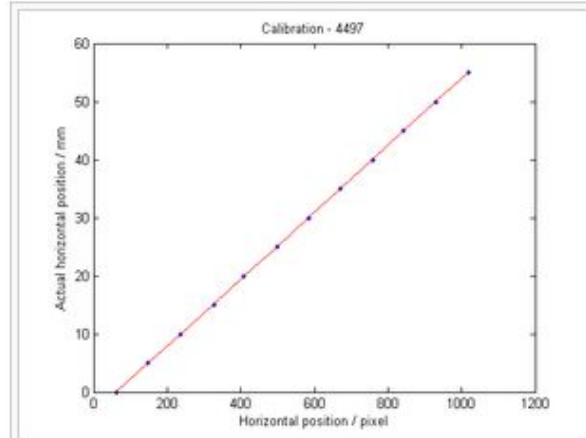
2013 Project

(Images from

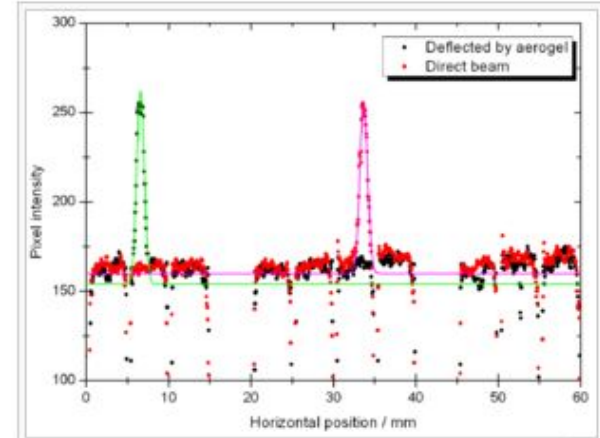
http://www.vsl.cua.edu/cua_phy/index.php/MainPage:Nuclear:KaonDetector:AerogelCharacteristics)



4) Calibration process example: a MatLAB code identifies the grid lines of the target paper.



5) Knowing the grid size of the previous image, the relation pixel to millimeter can be calibrated along the desired line.



6) After the calibration of the two lines (directed and refracted beams), the peaks position can be fitted for the analysis of the refracted beam deflection due to the aerogel.