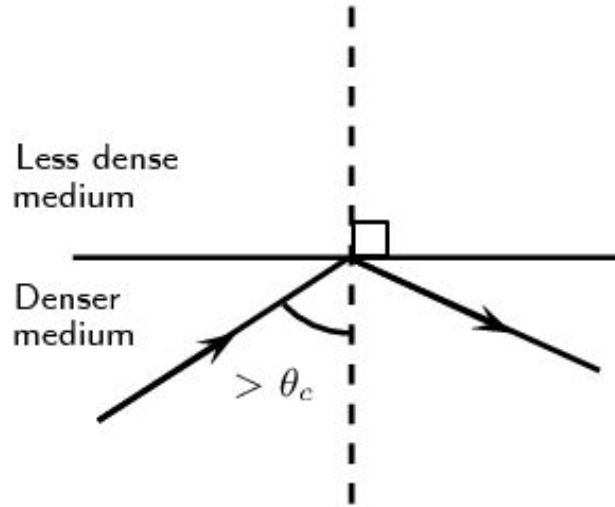
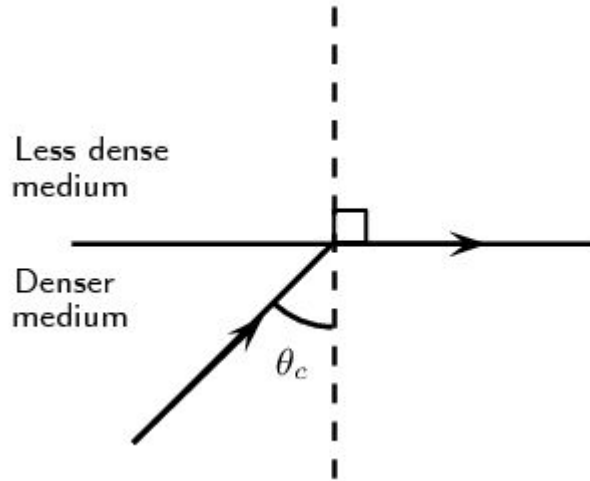


# The Critical Angle





# Index of Refraction (1)

$$\gamma = \alpha - \beta + \arcsin(n(\sin(\beta - \arcsin(\frac{\sin \alpha}{n}))))$$

$$n = \sqrt{\sin^2 \alpha + \sin^2(\gamma + \alpha)}$$

$\rightarrow \beta + d = 180^\circ \rightarrow d = 180^\circ - \beta$   
 $\rightarrow d = 180^\circ - (a + b)$   
 $\rightarrow \beta = a + b$   
 $\rightarrow \gamma = \theta_1 + \theta_2$   
 $\rightarrow \theta_1 = d - a$   
 $\rightarrow \theta_2 = c - b$   
 $\rightarrow \gamma = d - a + c - b$   
 $\rightarrow \gamma = d - (a + b) + c$   
 $\rightarrow \gamma = d - \beta + c$

$\beta$  not necessarily  $90^\circ$ , not necessarily  $= d$

$\rightarrow \frac{\sin \alpha}{\sin a} = \frac{n \sin c}{n \sin b}$   
 $\Rightarrow \frac{\sin \alpha}{\sin a} = \frac{\sin c}{\sin b}$   
 $\rightarrow \frac{\sin c}{\sin b} = \frac{n \sin c}{n \sin b}$   
 $\rightarrow$  Let  $\frac{n \sin c}{n \sin b} = n$   
 $\rightarrow c = \arcsin(n \sin(\beta - \arcsin(\frac{\sin \alpha}{n})))$   
 $\rightarrow \sin c = n \sin(\beta - \arcsin(\frac{\sin \alpha}{n}))$   
 $\rightarrow \sin b = \sin(\beta - \arcsin(\frac{\sin \alpha}{n}))$   
 $\rightarrow \sin a = \frac{\sin \alpha}{n}$   
 $\rightarrow a = \arcsin(\frac{\sin \alpha}{n})$

$$\rightarrow d = \gamma + c$$

$$\rightarrow \gamma = d - c$$

$$\rightarrow d - c + b = \gamma$$

$$\rightarrow \gamma = d + b - c$$

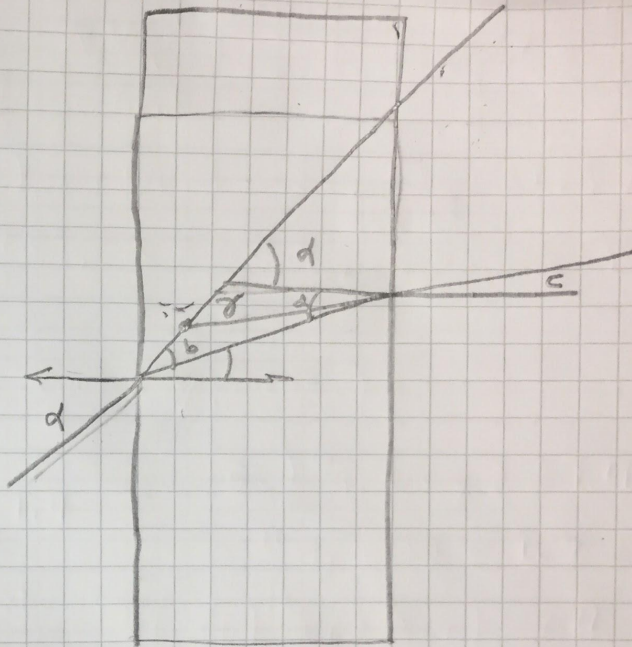
$$\rightarrow d = b + c$$

$$\rightarrow \frac{\delta n d}{\delta n (d-b)} = n = \frac{\delta n c}{\delta n d}$$

$$\rightarrow \frac{\delta n d}{\delta n c} = \frac{\delta n (d-b)}{\delta n d} = \frac{\delta n (b+d-b)}{\delta n d} = 1$$

$$\rightarrow d = c \quad \therefore \gamma = 0$$

*\therefore Why previous equation*

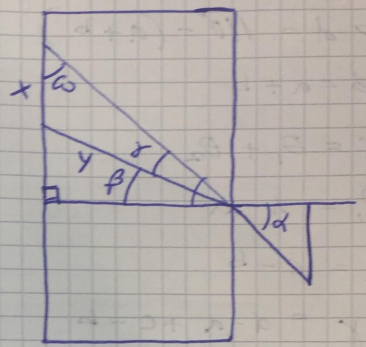


The index cannot be calculated using the first equation when the laser is shone through the body of the prism rather than across the angle

## Index of Refraction (2)

- $n = (\sin \alpha) / (\sin(\alpha - \arcsin(x \cos \alpha) / y))$
- $n = (\sin \alpha) / (\sin \beta)$

$\rightarrow \beta = \alpha - \gamma$   
 $\rightarrow \omega = 180 - \alpha - \beta$   
 $\rightarrow \omega = 90 - \alpha$



$\rightarrow \frac{y}{\sin \omega} = \frac{x}{\sin \gamma}$   
 $\rightarrow \gamma = \arcsin\left(\frac{(\sin \omega) x}{y}\right)$   
 $\rightarrow \beta = \alpha - \arcsin\left(\frac{(\sin \omega) x}{y}\right)$   
 $\rightarrow \frac{\sin \alpha}{\sin \beta} = \frac{n_{air}}{n_{prism}} = n$

$\rightarrow n = \frac{\sin \alpha}{\sin\left(\alpha - \arcsin\left(\frac{\sin(90 - \alpha) x}{y}\right)\right)}$

$\rightarrow n = \frac{\sin \alpha}{\sin\left(\alpha - \arcsin\left(\frac{x \cos(\alpha)}{y}\right)\right)}$

# Measurements and Equations (2)

[https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK\\_qlBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1868993415](https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK_qlBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1868993415)

- $n = (\sin \alpha) / (\sin(\alpha - \arcsin(x \cos \alpha) / y))$
- $n = (\sin \alpha) / (\sin \beta)$
- $\delta n(\alpha, x, y) = \sqrt{((\partial n / \partial \alpha) \delta \alpha)^2 + ((\partial n / \partial x) \delta x)^2 + ((\partial n / \partial y) \delta y)^2}$
- $\partial n / \partial \alpha =$

Handwritten derivation of  $\partial n / \partial \alpha$  on lined paper:

$$\rightarrow = \frac{\cos \alpha (\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})) - \sin \alpha (\cos(\alpha - \arcsin(\frac{x \cos \alpha}{y}))) (1 - \frac{x \sin \alpha}{y \sqrt{1 - (\frac{x \cos \alpha}{y})^2}})}{(\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})))^2}$$

Handwritten derivation of  $\partial n / \partial y$  on lined paper:

$$\rightarrow \frac{\partial n}{\partial y} = \frac{- \frac{x \cos \alpha}{y^2 (1 - (\frac{x \cos \alpha}{y})^2)^{\frac{3}{2}}} \sin(\alpha - \arcsin(\frac{x \cos \alpha}{y}))}{\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y}))}$$

Handwritten derivation of  $\partial n / \partial x$  on lined paper:

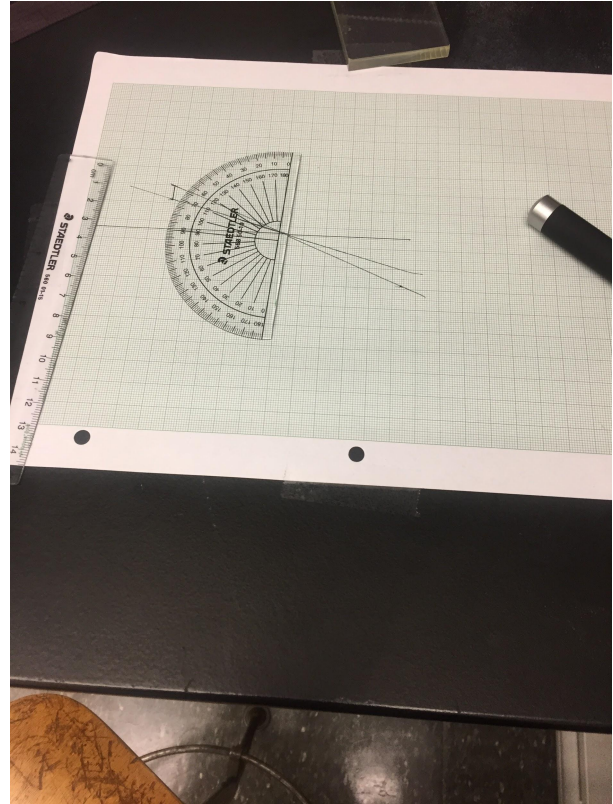
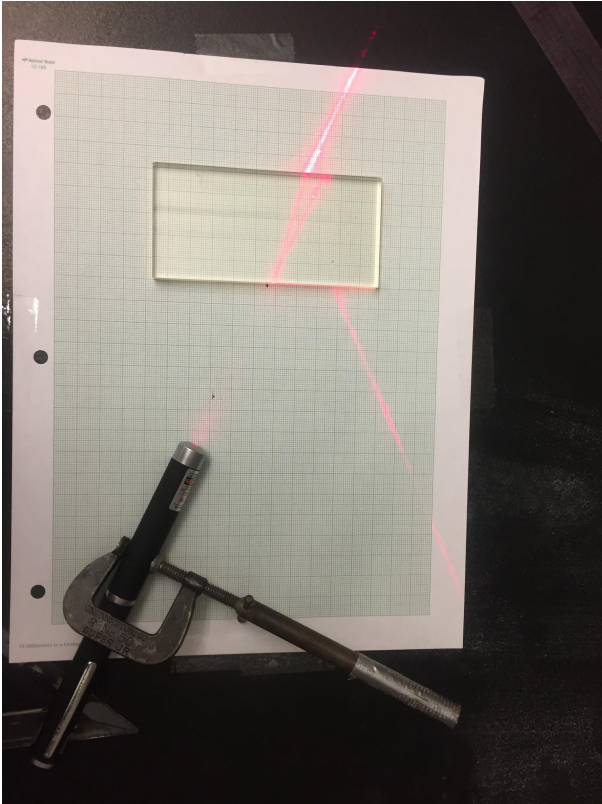
$$\rightarrow \frac{\partial n}{\partial x} = \frac{- \sin \alpha \cos \alpha}{y (1 - (\frac{x \cos \alpha}{y})^2)^{\frac{3}{2}} (\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})))}$$

# Original Measurement Approach

- Measuring while the laser was on
- Using original graph paper as reference
- Taking whole laser width as systematic error with additional random error
- Taking each measurement of 4 values as a set
- Problems:
  - Beam lifted up and put down to be able to see angles - eventually propped it up instead
  - Laser wouldn't stay on
  - Prism and laser easily inadvertently moved
  - Protractor placed on top of prism - additional error from perspective
  - Small protractor, beam covers multiple degrees
  - Beam refracted through transparent ruler and protractor
  - Estimated error of +/- .2 for n

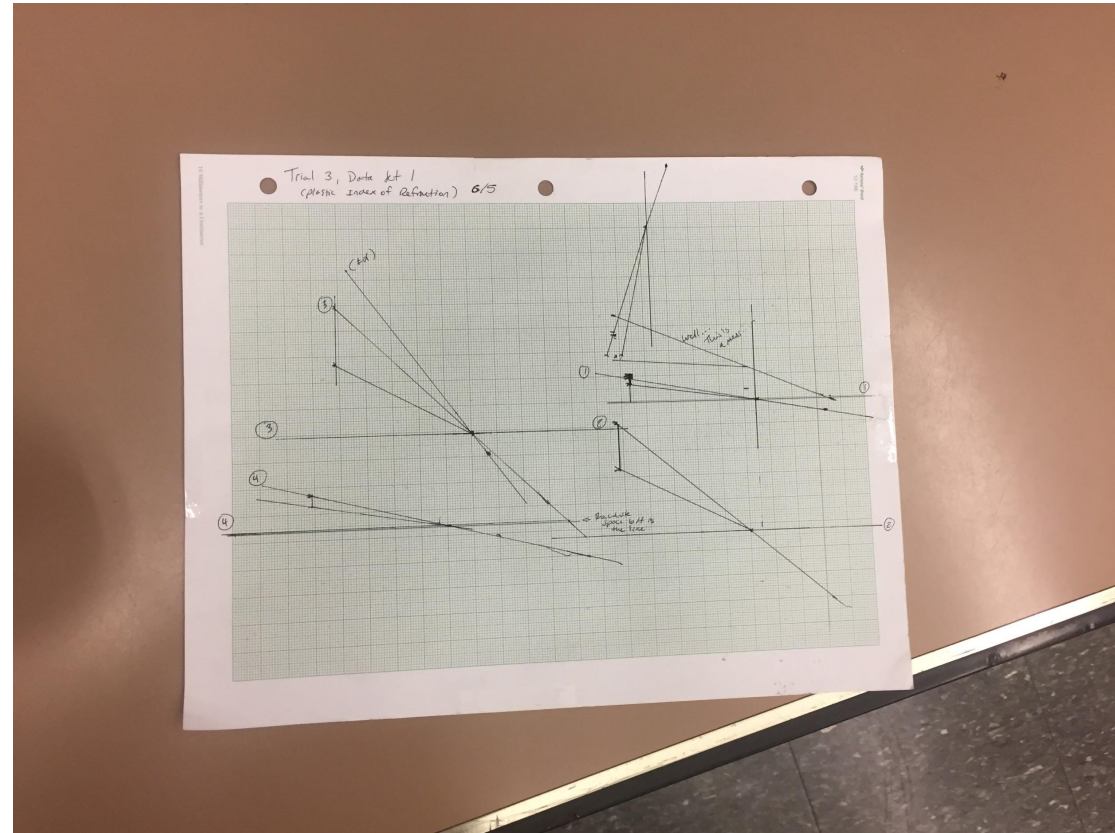


# New Approach



# New Approach

- Marking the location of the beam and measuring afterwards
- Using more precise graph paper
- Estimating the position of the edge of the laser beam (set 1) and then the center of the laser beam (set 2) as the point of measurement
- Taking each measured value independently
- Problems:
  - Protractor
  - Lines and points contribute to possible systematic error
  - Still error of .06+ for  $n$
  - Data doesn't agree  $\rightarrow$  error underestimated

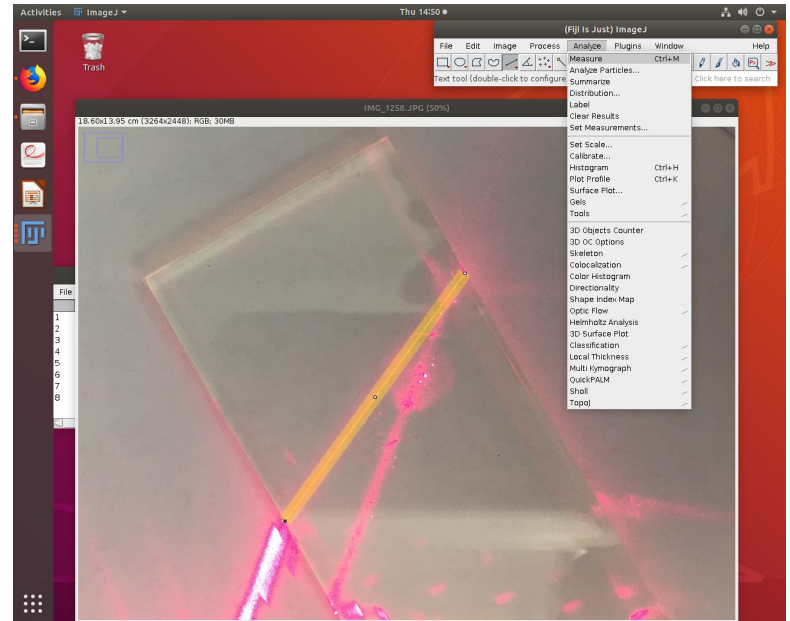
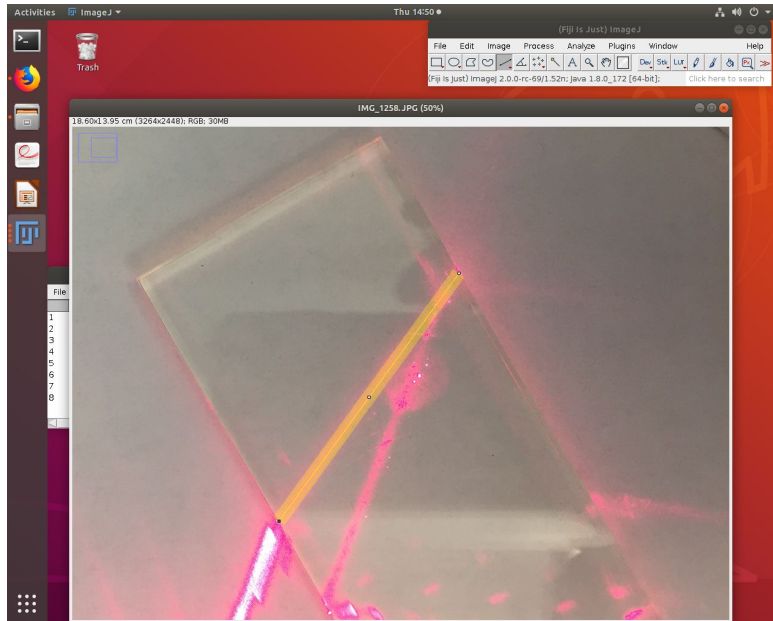


[https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK\\_qlBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1346628582](https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK_qlBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1346628582) - Spreadsheet

# Possible Image Analysis Software

- ImageJ - Free Image Analysis Software <https://imagej.nih.gov/ij/features.html>
  - “Measure area, mean, standard deviation. Measure lengths and angles. Use real world measurement units such as millimeters. Generate histograms and profile plots”
- Fiji - ImageJ Package <http://fiji.sc/>
  - Same measurement features as ImageJ - Tested this week
- Digimizer - not free <https://www.digimizer.com/features.php>
  - ‘Define unit of measurement, measure distances and lengths of line segments or path, measure angles, locate middle of line segment, measurements on perpendicular lines, fit line’
- Adobe Photoshop - <https://helpx.adobe.com/photoshop/using/measurement.html>
  - “Using the Photoshop Measurement feature you can measure any area defined with the Ruler tool or with a selection tool, including irregular areas selected with the Lasso, Quick Select, or Magic Wand tools. You can also compute the height, width, area, and perimeter, or track measurements of one image or multiple images”

# ImageJ





# ImageJ

