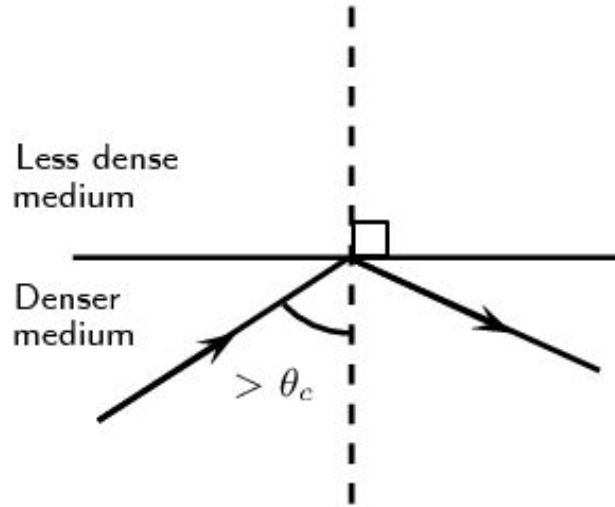
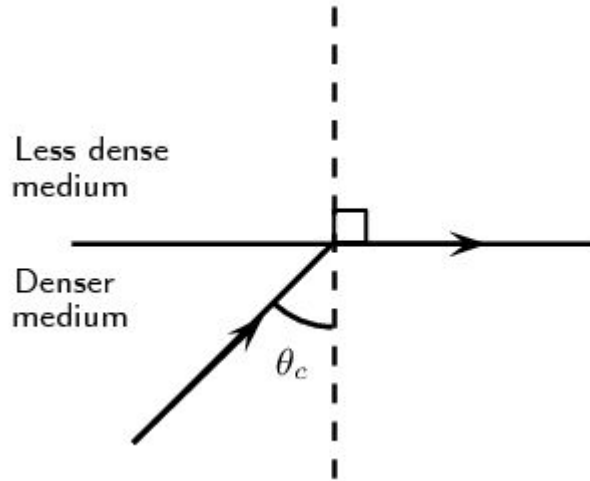


# The Critical Angle





# Index of Refraction (1)

$$\gamma = \alpha - \beta + \arcsin(n(\sin(\beta - \arcsin(\frac{\sin \alpha}{n}))))$$

$$n = \sqrt{(\sin^2 \alpha + \sin^2(\gamma + \alpha))}$$

$\rightarrow \beta + d = 180^\circ \rightarrow d = 180^\circ - \beta$   
 $\rightarrow d = 180^\circ - (a + b)$   
 $\rightarrow \beta = a + b$   
 $\rightarrow \gamma = \theta_1 + \theta_2$   
 $\rightarrow \theta_1 = d - a$   
 $\rightarrow \theta_2 = c - b$   
 $\rightarrow \gamma = d - a + c - b$   
 $\rightarrow \gamma = d - (a + b) + c$   
 $\rightarrow \gamma = d - \beta + c$

$\beta$  not necessarily  $90^\circ$ , not necessarily  $= d$

$\rightarrow \frac{\sin \alpha}{\sin a} = \frac{n \sin c}{n \sin b}$   
 $\Rightarrow \frac{\sin \alpha}{\sin a} = \frac{\sin c}{\sin b}$   
 $\rightarrow \frac{\sin c}{\sin b} = \frac{n \sin c}{n \sin b}$   
 $\rightarrow$  Let  $\frac{n \sin c}{n \sin b} = n$   
 $\rightarrow c = \arcsin(n \sin(\beta - \arcsin(\frac{\sin \alpha}{n})))$   
 $\rightarrow \sin c = n \sin(\beta - \arcsin(\frac{\sin \alpha}{n}))$   
 $\rightarrow \sin b = \sin(\beta - \arcsin(\frac{\sin \alpha}{n}))$   
 $\rightarrow \sin a = \frac{\sin \alpha}{n}$   
 $\rightarrow a = \arcsin(\frac{\sin \alpha}{n})$

$$\rightarrow d = \gamma + c$$

$$\rightarrow \gamma = d - c$$

$$\rightarrow d - c + b = \gamma$$

$$\rightarrow \gamma = d + b - c$$

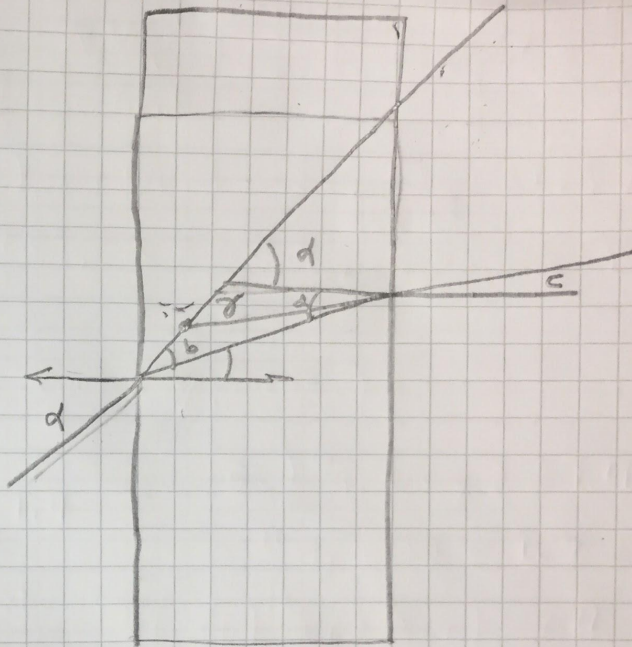
$$\rightarrow d = b + c$$

$$\rightarrow \frac{\delta m d}{\delta m (d - b)} = n = \frac{\delta m c}{\delta m d}$$

$$\rightarrow \frac{\delta m d}{\delta m c} = \frac{\delta m (d - b)}{\delta m d} = \frac{\delta m (b + d - b)}{\delta m d} = 1$$

$$\rightarrow d = c \quad \therefore \gamma = 0$$

*\therefore Why previous equation*

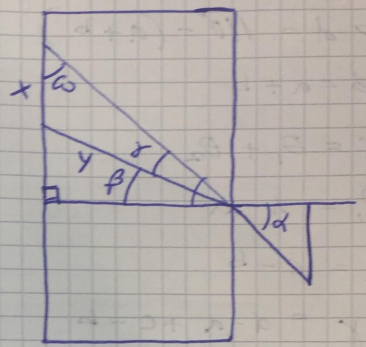


The index cannot be calculated using the first equation when the laser is shone through the body of the prism rather than across the angle

## Index of Refraction (2)

- $n = (\sin \alpha) / (\sin(\alpha - \arcsin(x \cos \alpha) / y))$
- $n = (\sin \alpha) / (\sin \beta)$

$\rightarrow \beta = \alpha - \gamma$   
 $\rightarrow \omega = 180 - \alpha - \beta$   
 $\rightarrow \omega = 90 - \alpha$



$\rightarrow \frac{y}{\sin \omega} = \frac{x}{\sin \gamma}$

$\rightarrow \gamma = \arcsin\left(\frac{(\sin \omega) x}{y}\right)$

$\rightarrow \beta = \alpha - \arcsin\left(\frac{(\sin \omega) x}{y}\right)$

$\rightarrow \frac{\sin \alpha}{\sin \beta} = \frac{n_{air}}{n_{prism}} = n$

$\rightarrow n = \frac{\sin \alpha}{\sin\left(\alpha - \arcsin\left(\frac{\sin(90 - \alpha) x}{y}\right)\right)}$

$\rightarrow n = \frac{\sin \alpha}{\sin\left(\alpha - \arcsin\left(\frac{x \cos(\alpha)}{y}\right)\right)}$

# Measurements and Equations (2)

[https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK\\_qlBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1868993415](https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK_qlBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1868993415)

- $n = (\sin \alpha) / (\sin(\alpha - \arcsin(x \cos \alpha) / y))$
- $n = (\sin \alpha) / (\sin \beta)$
- $\delta n(\alpha, x, y) = \sqrt{((\partial n / \partial \alpha) \delta \alpha)^2 + ((\partial n / \partial x) \delta x)^2 + ((\partial n / \partial y) \delta y)^2}$
- $\partial n / \partial \alpha =$

Handwritten derivation of  $\partial n / \partial \alpha$  on lined paper:

$$\rightarrow = \frac{\cos \alpha (\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y}))) - \sin \alpha (\cos(\alpha - \arcsin(\frac{x \cos \alpha}{y}))) (1 - \frac{x \sin \alpha}{y \sqrt{1 - (\frac{x \cos \alpha}{y})^2}})}{(\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})))^2}$$

Handwritten derivation of  $\partial n / \partial y$  on lined paper:

$$\rightarrow \frac{\partial n}{\partial y} = \frac{- \frac{x \cos \alpha}{y^2 (1 - (\frac{x \cos \alpha}{y})^2)^{\frac{3}{2}}} \sin(\alpha - \arcsin(\frac{x \cos \alpha}{y}))}{\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y}))}$$

Handwritten derivation of  $\partial n / \partial x$  on lined paper:

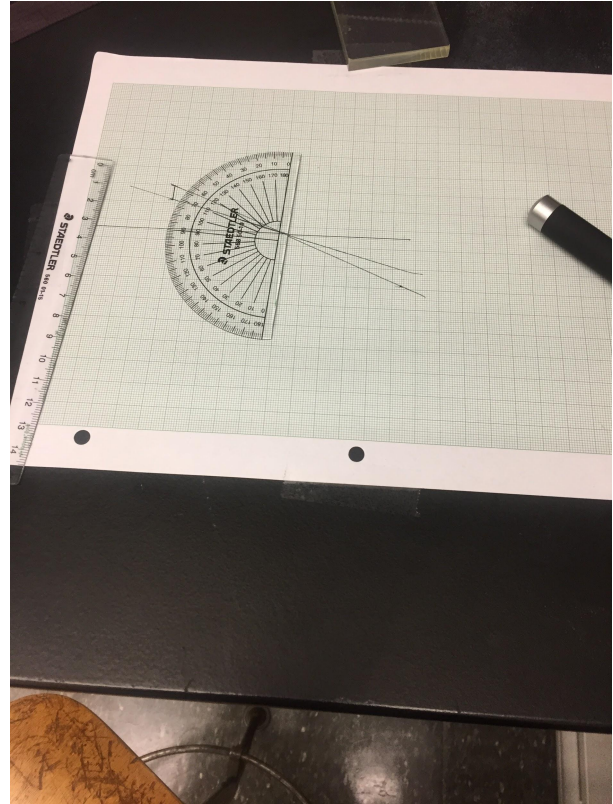
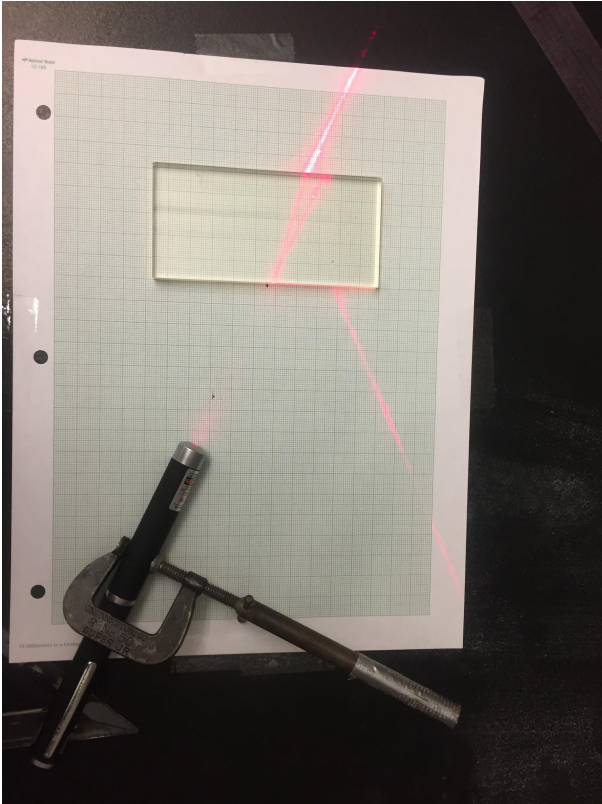
$$\rightarrow \frac{\partial n}{\partial x} = \frac{- \sin \alpha \cos \alpha}{y (1 - (\frac{x \cos \alpha}{y})^2)^{\frac{3}{2}} (\sin(\alpha - \arcsin(\frac{x \cos \alpha}{y})))}$$

# Original Measurement Approach

- Measuring while the laser was on
- Using original graph paper as reference
- Taking whole laser width as systematic error with additional random error
- Taking each measurement of 4 values as a set
- Problems:
  - Beam lifted up and put down to be able to see angles - eventually propped it up instead
  - Laser wouldn't stay on
  - Prism and laser easily inadvertently moved
  - Protractor placed on top of prism - additional error from perspective
  - Small protractor, beam covers multiple degrees
  - Beam refracted through transparent ruler and protractor
  - Estimated error of +/- .2 for n

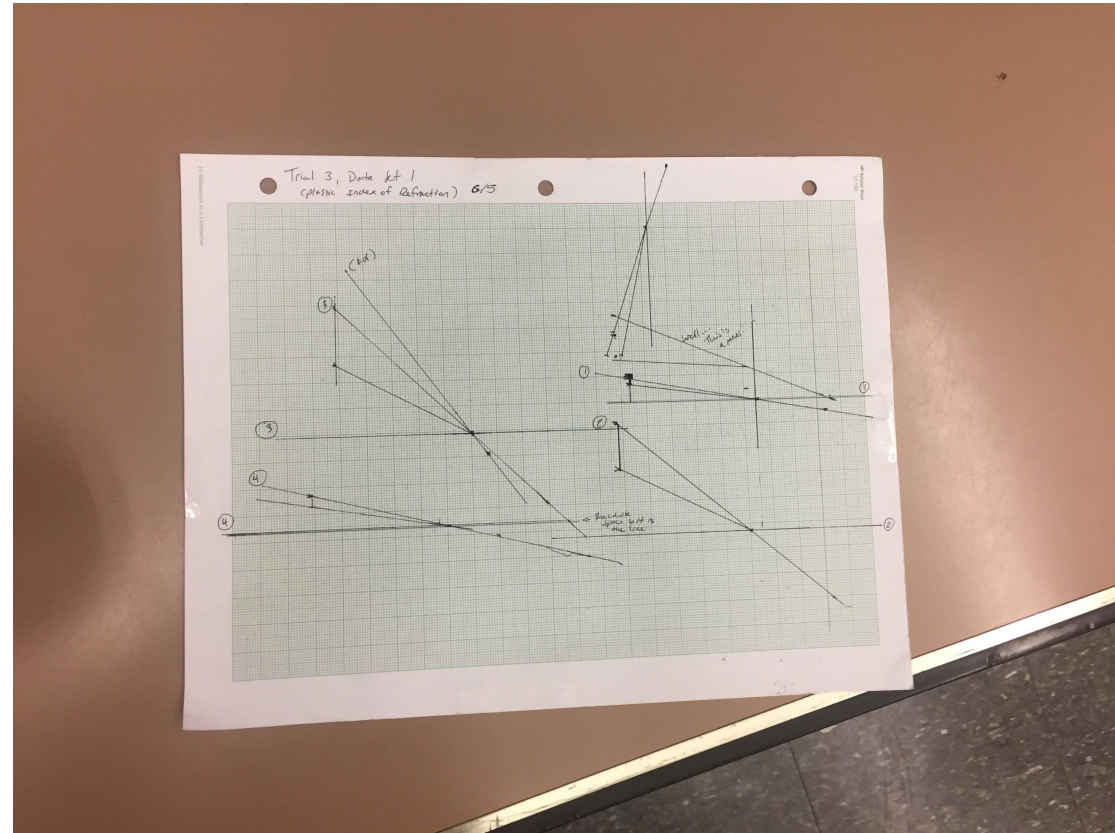


# New Approach



# New Approach

- Marking the location of the beam and measuring afterwards
- Using more precise graph paper
- Estimating the position of the edge of the laser beam (set 1) and then the center of the laser beam (set 2) as the point of measurement
- Taking each measured value independently
- Problems:
  - Protractor
  - Lines and points contribute to possible systematic error
  - Still error of .06+ for  $n$
  - Data doesn't agree -> error underestimated



# New Approach

[https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK\\_glBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1346628582](https://docs.google.com/spreadsheets/d/1Txx-vV3wgf3GK_glBFUmlLhNHlCuCy1psQFxxUU7tCow/edit#gid=1346628582)