Hunting for the Source of Visible Mass

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One hundred years ago, the only matter particles known to exist were the proton, neutron, and electron; but with the march of discovery, the advent of cosmic ray science and particle accelerators, numerous additional particles were discovered, *e.g.* muon (1937), pion (1947), kaon (1947), … . By the latter half of the twentieth century, it was widely held that not all the particles could be elementary: a new paradigm was thought to be necessary and vigorously sought. A crucial advance was made with the Constituent-Quark Theory, proposed independently by Gell-Mann [1] and Zweig [2], for which Gell-Mann was awarded the 1969 Nobel Prize in Physics: "for his contributions and discoveries concerning the classification of elementary particles and their interactions".

The next thirty years were a period of intense activity that created the Standard Model of particle physics, which lays out the basics of how elementary particles and forces interact in the universe. Critically, however, the theory fails to explain how particles actually get their mass. The Higgs boson, whose discovery in 2012 completed the array of seventeen elementary particles within the Standard Model – electrons, quarks, *etc.*, is often said to give mass to everything. However, that is wrong. The Higgs only gives mass to some very simple particles, accounting for just one or two percent of the mass of more complex things like atoms, molecules and everyday objects.

At this point it is right to ask a question; namely, what is mass? Einstein argued that energy is mass; and mass is the essence of matter. The burning issue, therefore, is the need to expose the nature of "Matter" in the Standard Model: where does it obtain its mass; what is the origin of its interaction scale; how is it built from the elementary degrees-of-freedom used to define the Standard Model?

Atomic nuclei lie at the core of everything we can see; and at the first level of approximation, their atomic weights are simply the sum of the masses of all the neutrons and protons they contain. Each neutron and proton has a mass of $m_N \approx 1\,\text{GeV}$, *i.e.* approximately 2000-times the electron mass, m_e. The Higgs boson produces m_e, but what produces the masses of the neutron and proton? This is the crux: the vast majority of the mass of a neutron or proton is lodged with the energy needed to hold quarks together inside them; and that is supposed to be explained by quantum chromodynamics (QCD), the strong-interaction piece within the Standard Model.

QCD is the first theory to demand that science fully resolve the conflicts generated by joining relativity and quantum mechanics. In attempting to match QCD with Nature, it is necessary to confront the innumerable complexities of strong, nonlinear dynamics in relativistic quantum field theory, *e.g.* the loss of particle number conservation, the frame and scale dependence of the explanations and interpretations of observable processes, and the evolving character of the relevant degrees-of-freedom. Electroweak theory and phenomena are essentially perturbative, possessing none of this complexity.

It has become apparent that QCD is unique. It is the only known fundamental theory with the capacity to sustain massless elementary degrees-of-freedom, *viz.* gluons and quarks. Yet gluons and quarks are predicted to acquire mass dynamically [3, 4, 5] so that the only massless systems in QCD are its composite Nambu-Goldstone bosons. All other everyday bound states possess nuclear-size masses, far in excess of anything that can directly be tied to the Higgs boson. Science has never before encountered an interaction such as that at work in QCD; and charting this interaction, explaining and understanding everything of which it is capable can potentially change the way we look at the Universe. In fact, it may be that QCD establishes the paradigm which will truly enable us to step beyond the Standard Model.

In QCD, the interaction is everything; and yet the Lagrangian is remarkably simple in appearance:

$$\mathcal{L}_{\rm QCD} = \sum_j \bar{q}_j [i\gamma^\mu D_\mu - m_j] q_j - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} , \tag{1a}$$

$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu , \quad G^a_{\mu\nu} = \partial_\mu A^a_\nu \partial_\nu A^a_\mu + \boxed{ig f^{abc} A^b_\mu A^c_\nu} , \tag{1b}$$

where $\{q_j\}$ are the quark fields, with j their flavour label and m_j their Higgs-generated current-quark masses, $\{A^a_\mu\}$ are the gluon fields, and $\{\frac{1}{2}\lambda^a\}$ are the generators of the (colour/chromo) gauge-group $SU(3)$ in the fundamental representation. In comparison with quantum electrodynamics (QED), the single, essential difference is the term describing gluon self-interactions, contained within the box. If QCD is correct, as suggested strongly by its ability to describe and predict high-energy phenomena for which the theory is perturbative owing to asymptotic freedom, then this term must hold the answers to an enormous number of Nature's basic questions, *e.g.*: what is the origin of visible mass and how is it distributed within atomic nuclei; and what carries the proton's spin and how can the same degrees-of-freedom combine to ensure the pion is spinless?

Indeed, at the most fundamental level, the existence of the known Universe depends critically on the following empirical facts: the proton is massive, *i.e.* the mass-scale for strong interactions is vastly different to that of electromagnetism; it is absolutely stable, despite being a composite object constituted from three valence quarks; and the pion, the primary carrier of the force within nuclei, is unnaturally light, with a lepton-like mass, despite being a strongly interacting composite object built from a valence-quark and valence-antiquark. Nowhere are there more basic expressions of *emergence* in Nature, *viz.* low-level rules, with evident simplicity, Eqs. (1), producing high-level phenomena, with enormous apparent complexity.

Treated as a classical theory, chromodynamics is a non-Abelian local gauge field theory. As with all such theories formulated in four spacetime dimensions, no mass-scale exists in the absence of Lagrangian masses for the quarks. There is no dynamics in a scale-invariant theory, only kinematics: the theory looks the same at all length-scales and hence there can be no clumps of anything. Bound-states are therefore impossible and, accordingly, our Universe cannot exist. As highlighted above, spontaneous symmetry breaking, as realised via the Higgs mechanism, does not solve this problem because normal matter is constituted from light-quarks, u (up) and d (down), and the masses of the neutron and proton, the kernels of all visible matter, are roughly 100-times larger than anything the Higgs can produce in connection with u- and d-quarks.

No amount of staring at $\mathcal{L}_{\rm QCD}$ can reveal the 1 GeV mass-scale that characterises atomic nuclei. This presents a stark contrast with QED, in which the spectrum of hydrogen energy levels is measured by m_e and the fine-structure constant, both of which appear explicitly in that theory's Lagrangian. Consequently, the question of how did the Universe come into being is inseparable from the questions of how does a mass-scale appear and why does it have the value that is observed?

In field theory, scale invariance is expressed in conservation of the dilation current: $\partial_\mu \mathcal{D}_\mu = T_{\mu\mu} = 0$, where $T_{\mu\nu}$ is the theory's energy-momentum tensor. The catastrophic consequences of scale invariance are avoided in Nature through the agency of quantum effects. In quantising QCD, regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale. This is "dimensional transmutation": mass-dimensionless quantities become dependent on a mass-scale, and this entails the appearance of a "trace anomaly":

$$T_{\mu\mu} = \beta(\alpha(\zeta))\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} =: \Theta_0\,, \tag{2}$$

where $\beta(\alpha)$ is QCD's β-function, which describes how the coupling runs, ζ is the renormalisation scale, and this equation assumes the chiral-limit, *i.e.* the Higgs mechanism is turned-off for quarks.

Simply knowing that a trace anomaly exists does not deliver much: it only indicates that there is a mass-scale. The crucial issue is whether or not one can compute and/or understand the magnitude of that scale. One can certainly measure the size of the QCD scale anomaly, for consider the expectation value of the energy-momentum tensor in the proton: $\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$. In the chiral limit, it follows that

$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_N^2 = \langle p(P)|\Theta_0|p(P)\rangle\,; \tag{3}$$

namely, the entirety of the proton mass is produced by gluons: the trace anomaly is measurably large; and that property must logically owe to gluon self-interactions, which are also responsible for asymptotic freedom.

It is important to note that all renormalisable four-dimensional theories possess a trace anomaly. Hence, in this, QED and QCD are alike. However, years of comparing systems bound by electromagnetism with those produced by the strong interaction indicate that the size of the trace anomaly in QED must be a great deal smaller than that in QCD. This disparity may be understood by noting that, with reference to the generating functional for one-particle irreducible Schwinger functions,

$$\int d^4x\, G_{\mu\nu}G_{\mu\nu} \sim \int d^4x d^4y\, A_\mu(x) D^{-1}_{\mu\nu}(x-y) A_\nu(y)\,, \tag{4}$$

where $D^{-1}_{\mu\nu}$ is the fully-dressed gauge-boson 2-point function (dressed propagator). Eq. (4) shows that if any mass-scale is to become associated with the trace anomaly, then it will be exhibited in the gauge-boson vacuum polarisation. Textbooks show that the photon vacuum polarisation does not possess an infrared mass-scale: $\Pi_{\rm QED}(k^2=0)=0$, so that it serves merely to produce the very slow running of the QED coupling, *i.e.* any dynamical violation of the conformal features of QED are very small and hence the trace anomaly is negligible. In contrast, owing to gauge sector dynamics, a Schwinger mechanism is active in QCD [3, 4], so that

$$k^2\Pi_{\rm QCD}(k^2)\big|_{k^2=0} \sim (m_N/2)^2 \tag{5}$$

and the QCD trace anomaly expresses a mass-scale that is, empirically, very significant.

There is a flip-side to Eq. (3), *viz.* one can replace the proton state by the pion:

$$\langle \pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \Rightarrow \langle \pi(q)|T_{\mu\mu}|\pi(q)\rangle \stackrel{\text{chiral limit}}{=} 0\,, \tag{6}$$

because the pion is a massless Nambu-Goldstone mode. Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e.* each term in the expression of the operator vanishes when evaluated in the pion and thus gluons contribute nothing to the pion mass? The answer is *no*. Instead [6], Eq. (6) owes to cancellations between different operator-component contributions; and the cancellation is exact in the pion channel because of dynamical chiral symmetry breaking (DCSB).

The combination of Eqs. (3) and (6) expresses a peculiar dichotomy, which insists that no answer to the question "Whence the proton's mass?" is complete unless it simultaneously solves the additional puzzle "Whence the *absence* of a pion mass?" The natural nuclear-physics mass-scale, m_N, must emerge simultaneously with apparent preservation of scale invariance in an intimately related system. Furthermore, in the chiral limit, $\langle\pi(q)|\Theta_0|\pi(q)\rangle \equiv 0$ irrespective of the size of m_N.

These statements hold with equal force on a sizeable neighbourhood of the chiral limit because hadron masses are continuous functions of the current-quark masses. Thus, given the Gell-Mann–Oakes–Renner relation [7], it follows that the entirety of the mass of the physical pion owes to the current-quark mass term in QCD's Lagrangian. Indeed, for any meson, M,

$$s_M(0) = \langle M(q)|m\bar{q}q|M(q)\rangle = m\frac{\partial m_M^2}{\partial m}\,, \tag{7}$$

viz. the meson's scalar form factor at zero momentum transfer measures the response of its mass-*squared* to a change in current-quark mass [8]. Notably, the pion (and any other Nambu-Goldstone mode) possesses the peculiar property that

$$s_\pi(0) \stackrel{m\simeq 0}{=} m\frac{\partial m_\pi^2}{\partial m} = 1\times m_\pi^2, \tag{8}$$

which is a restatement of the fact that in the neighbourhood of the chiral limit, 100% of the pion mass-squared owes to the current-mass in $\mathcal{L}_{\rm QCD}$. It is natural to compare this result with that for the pion's spin-flip partner, *i.e.* the ρ-meson [9]: $s_\rho(0)\approx 0.06\,m_\rho^2$, indicating that just 6% of the ρ-meson's mass-squared is generated by the current-mass term in $\mathcal{L}_{QCD}$.

The key to understanding Eq. (6) is a quartet of Goldberger-Treiman-like (GT) relations [10], the best known of which states:

$$f_\pi E_\pi(k;0) \stackrel{m\simeq 0}{=} B(k^2)\,, \tag{9}$$

where f_π is the pion's leptonic decay constant, E_π is the leading piece of its Bethe-Salpeter amplitude, and B is the scalar piece of the dressed-quark self-energy. Eq. (9) and its partners are exact in chiral QCD and they express the fact that Goldstone's theorem is fundamentally an expression of equivalence between the quark one-body problem and the two-body bound-state problem in QCD's colour-singlet pseudoscalar channel. It follows that, enigmatically, the properties of the nearly-massless pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the Universe. It is notable that a rudimentary form of Eq. (9) can be found in Ref. [11], the work which brought Nambu one half of the 2008 Nobel Prize in Physics for "the discovery of the mechanism of spontaneous broken symmetry in subatomic physics".

With this quartet of GT relations in hand, one can construct an algebraic proof [10, 12] that at any and each order in a symmetry-preserving truncation of those equations in quantum field theory that are necessary to describe a pseudoscalar bound state, there is a precise cancellation between the mass-generating effect of dressing the valence-quarks which constitute the system and the attraction generated by the interactions between them. This cancellation guarantees that the seed two-parton system, which began massless, becomes a complex system, with a nontrivial bound-state wave function attached to a pole in the scattering matrix that is located at $P^2 = 0$, *viz.* Eq. (6) is obtained through cancellations between one-body dressing and two-body interaction effects that sum precisely to zero because chiral symmetry is dynamically broken:

$$\text{pion:}\quad M_{\text{dressed quark}} + M_{\text{dressed antiquark}} + U_{\text{binding between constituents}} \approx 0\,. \tag{10}$$

Since QCD's interactions are universal, *i.e.* they are the same within all hadrons, then what is true for the pion must also be true for the proton, *etc.* Unsurprisingly, therefore, all studies of the coupled quantum field equations which describe the proton reveal that the same cancellations take place within the proton. However, in this channel there is no symmetry that requires the cancellations

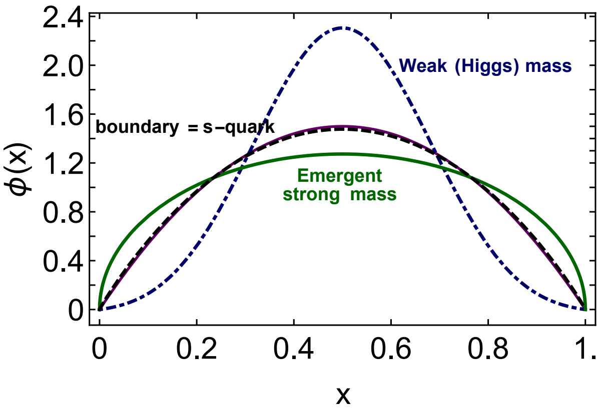

Figure 1: Twist-two PDAs computed at a resolving scale $\zeta = 2\,\text{GeV}$. Solid (green) curve – pion; dot-dashed (blue) curve – η_c; solid (thin, purple) curve – conformal limit result, $6x(1-x)$; and dashed (black) curve – "heavy-pion", *i.e.* a pion-like pseudoscalar meson in which the valence-quark current-masses take *s*-quark values.

to be complete and hence the proton's mass has a value that is typical of the magnitude of scale breaking in QCD's one body sectors, *viz.* the dressed-gluon and -quark mass-scales. A clear analogy with quantum mechanics finally emerges: the mass of a QCD bound-state is the sum of the masses of the constituents plus some (negative) binding energy.

The picture just described may be described as the "DCSB paradigm". It provides a basis for understanding why the mass-scale for strong interactions is vastly different to that of electromagnetism, why the proton mass expresses that scale, and why the pion is nevertheless unnaturally light. What, then, of the absolute stability of the proton, *i.e.* why does the proton not decay into three isolated quarks? The answer to this question is an explanation of confinement, which no one can yet claim to provide. The DCSB paradigm does suggest a line or reasoning, however. Classical chromodynamics is a non-Abelian local gauge theory. Consequently, the concept of local gauge invariance persists, but there is no confinement without a mass-scale. For example, three quarks can be prepared in a colour-singlet combination and colour rotations will keep the three-body system colour neutral, but the quarks involved need not have any proximity to one another. Indeed, proximity is meaningless because all lengths are equivalent in a scale invariant theory. Hence, the question of "Whence mass"? is equivalent to "Whence a mass-scale?", which is equivalent to "Whence a confinement scale?". Thus, understanding the origin, Eq. (3), and absence, Eq. (6), of mass in QCD is quite likely inseparable from the task of understanding confinement. It is probable, therefore, that confinement and DCSB are intimately linked owing to the prominent role of light-quarks in real-world QCD, and confinement is a dynamical, emergent phenomenon, as canvassed in Ref. [5], Sec. 3.

It is now worth highlighting another implication of the DCSB paradigm. Asymptotic freedom means that the ultraviolet behaviour of QCD is controllable. The only other thing that can "go wrong" with a quantum field theory is the appearance of infrared divergences. However, owing to the particular nature and strength of gauge-sector interactions in QCD, mass-scales are dynamically generated for both gluons and quarks, *viz.* QCD generates its own infrared cutoffs [13]. The complete range of consequences of this effect is just beginning to be explored, but the possibility that QCD might therefore be the first known example of a completely consistent quantum field theory is prominent amongst them. Should that be true, then QCD presents itself as a strong candidate upon which to base realistic extensions of the Standard Model.

At this point it is apparent that no significant mass-scale is possible in QCD unless one of commensurate size is expressed in the dressed-propagators of gluons and quarks. It follows that the mechanism(s) responsible for the generation of mass in QCD can be exposed by measurements that are sensitive to such dressing effects. As an example, consider a particular class of meson "wave functions",

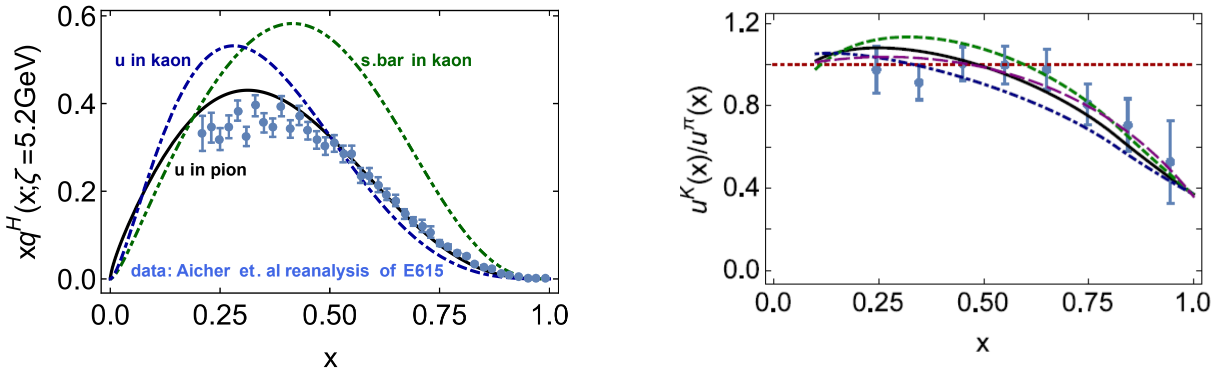

Figure 2: *Left panel* – Valence-quark distribution functions computed for the pion and kaon [15]. The disparity between the magnitudes of the u-quark-in-pion PDF and those for the kaon's valence quarks shows clearly that much more of the pion's momentum is carried by other bound-state participants, especially gluons. (Data in the left panel are from Ref. [16], rescaled according to the reanalysis described in Ref. [17].) *Right panel* – Solid (black) curve, obtained via LO evolution from $\zeta_H = 0.51\,\text{GeV}$, assuming 5% of the kaon's momentum is carried by glue at this hadronic resolving scale; dashed (green) curve, zero momentum carried by gluons; and dot-dashed (blue) curve, 10% of the kaon's momentum carried by glue. In contrast, an analysis of πN Drell-Yan data suggests that 29% of the pion's momentum is carried by glue at ζ_H, *viz.* roughly six-times more glue than in the kaon. The long-dashed (purple) curve is the prediction in Ref. [18], obtained using numerical solutions of realistic gap and Bethe-Salpeter equations. (Data in the right panel are from Ref. [19]. The dotted (red) line marks a value of unity for the ratio. It is drawn to highlight the domain upon which one might be confident empirically that $u^K(x)/u^\pi(x) \neq 1$, *viz.* $x \gtrsim 0.7$.)

i.e. twist-two parton distribution amplitudes (PDAs), a number of which are depicted in Fig. 1. This image answers the following question: When does the Higgs mechanism begin to influence mass generation? In the limit $m_{\text{quark}} \to \infty$; namely, when the Higgs mechanism has overwhelmed every other mass generating force, $\varphi(x) = \delta(x - 1/2)$. Evidently, the η_c meson, constituted from a valence charm-quark and its antimatter partner, feels the Higgs mechanism strongly. On the other hand, $\varphi_\pi(x)$ is a broad, concave function. Such features are a definitive signal that pion properties express emergent mass generation [14]. The remaining example in Fig. 1 shows that the PDA for a system composed of s-quarks almost matches that of QCD's conformal (scale-free) limit, *viz.* this system lies at the boundary, with strong (emergent) mass generation and the weak (Higgs) mass playing an equal role. It follows that comparisons between distributions of truly light quarks and those describing s-quarks are ideally suited to exposing measurable signals of dynamical mass generation.

A striking example can be found in a comparison between the valence-quark distribution functions of the pion ($\pi^- = \bar{u}d$) and kaon ($K^- = \bar{u}s$). Experimental data on π and K PDFs are available, but they are old and sparse, having been obtained in mesonic Drell-Yan scattering from nucleons in heavy nuclei, with information on the pion's PDFs reported in Refs. [16, 20, 21] and results for the ratio of kaon and pion distribution functions presented in Ref. [19]. Newer data are necessary, owing to persistent doubts about the large Bjorken-x behaviour of the pion's valence-quark PDF [22]; because a single modest-quality measurement of the kaon-to-pion ratio cannot be considered definitive; and, most critically, because:

> QCD theory predicts [15] that the gluon content of the pion is vastly greater than that of the kaon, Fig. 2, in an expression of the near-complete cancellation between strong-mass generating dressing of the valence-quark and -antiquark, and binding attraction in the almost massless pion *cf.* the massive kaon, *i.e.* Eqs. (6), (10).

This prediction is an organic expression of the pion's appearance and role in the Standard Model as a Nambu-Goldstone boson. It means that existing textbook descriptions of Goldstone's theorem via pointlike modes is outdated and simplistic. Nambu-Goldstone modes in the Standard Model are far more interesting because they are:

- nonpointlike;
- intimately connected with the origin of mass;
- and, quite probably, play an fundamental role in the expression of confinement.

Empirical validation of this key prediction of the DCSB paradigm will enable science to write a new, definitive chapter in future textbooks on the Standard Model.

As highlighted by Fig. 2, the exaggerated gluon content of the Nambu-Goldstone pion is directly measurable in a comparison between the valence u-quark distributions in the π and K; and as we shall now explain, measurements of forward nucleon structure functions at an electron ion collider can provide the ideal means by which to empirically expose and validate this fascinating and critical aspect of emergent mass generation within the Standard Model.

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